Q.P Code **D 123240**	Total Pages **3**	Name **604818**
		Register No.

SECOND SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2025

MATHEMATICS

MAT2MN105 Vector Spaces and Linear Transformations

2024 Admission Onwards

Maximum Time :2 Hours | **Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	Let V be a vector space, $\mathbf{u}$ a vector in V, and k a scalar ; then: prove that $-1\mathbf{u} = -\mathbf{u}$
2	Show that set of all polynomials of the form $a_0 + a_1x + a_2x^2 + a_3x_3$ in which a_0, a_1, a_2 , and a_3 are rational numbers is not a subspace of $\mathbf{P_3}$
3	Define the terms (a) Linear combination of vectors (b) Span of vectors
4	Show that the functions $1, x, e^x$ are linearly independent vectors.
5	In any vector space a set that contains the zero vector must be linearly dependent. Explain why this is so.
6	Use matrix multiplication to find the reflection of (a, b) about the line $y = x$
7	Let $T_1 : R^2 \to R^2$ is the reflection about the x-axis, and $T_2 : R^2 \to R^2$ is the reflection about the line $y = x$. Determine whether the operators T_1 and T_2 commute; that is, whether $T_1 \circ T_2 = T_2 \circ T_1$.
8	Determine whether the following stated operators commute. "A reflection about the line $y = x$ and an expansion in the $x-$ direction with factor 2."
9	Let eigen values of a 4×4 matrix A be 1,2,3 and 4. Is A invertible? If so, find eigen values of A^{-1}
10	Is the matrix $\begin{bmatrix} 2 & 0 \\ 7 & 4 \end{bmatrix}$ diagonalizable? Why?

604818

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))

11	Let $V = R^2$ and define addition and scalar multiplication operations as follows: If $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$, then define $\mathbf{u}+\mathbf{v} = (u_1+v_1, u_2+v_2)$ and if k is any real number, then define $k\mathbf{u} = (ku_1, 0)$. Prove or Disprove that V is a vector space
12	Express the vector $6+11x+6x^2$ as a linear combination of $p_1 = 2+x+4x^2$, $p_2 = 1-x+3x^2$, and $p_3 = 3+2x+5x^2$.
13	Determine whether the matrices $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ are linearly independent or are linearly dependent in $\mathbf{M_{22}}$
14	Find a basis for the solution space of the following homogeneous linear system, and find the dimension of that space. $x_1 - 3x_2 + x_3 = 0$ $2x_1 - 6x_2 + 2x_3 = 0$ $3x_1 - 9x_2 + 3x_3 = 0$
15	(a) Use matrix multiplication to find the contraction of $(2, -1, 3)$ with factor $k = 1/4$ (b) Use matrix multiplication to find the dilation of (2, -1, 3) with factor $k = 2$.
16	Let $T_1(x_1, x_2, x_3) = (4x_1, -2x_1+x_2, -x_1-3x_2)$ and $T_2(x_1, x_2, x_3) = (x_1+2x_2, -x_3, 4x_1-x_3)$. (a) Find the standard matrices for T_1 and T_2 . (b) Find the standard matrices for $T_2 \circ T_1$ and $T_1 \circ T_2$
17	Let $A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Confirm that P diagonalizes A , and then compute each of the following powers of A. Also find A^{1250}
18	Find the eigenvalues and the corresponding eigenspaces of the stated matrix operator on R^2 . (a) Dilation with factor $k(k > 1)$. (b) Orthogonal projection onto the x- axis

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19	In each part, find a basis for the given subspace of R^3, and state its dimension. (a) The plane $3x - 2y + 5z = 0$. (a) The plane $x - y = 0$. (a) All vectors of the form (a, b, c), where $b = a + c$
20	Find bases for the eigenspaces of $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$