D 114586 **(Pages : 3)** Name..

Reg. No....................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that for all elements x and y of X, $x \cdot x = x$.
2. Illustrate concise table for a Boolean function through an example.
3. Define Lattice.
4. If G is a simple graph and $\delta \geq k$, then prove that G contains a path of length at least k.
5. Give examples of vertex cuts.
6. If the girth k of a connected plane graph G is at least 3, then prove that $m \leq \frac{k(n-2)}{(k-2)}$.
7. Define dfa. Give an example.
8. Define nfa. Give an example.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Define a chain in a poset. Prove that the intersection of two chains is a chain.

10. State and prove De Morgan's laws in a Boolean algebra.

11. Write the following Boolean function in their disjunctive normal form.

$$f\left(x_1, x_2, x_3\right) = \left(x_1 + x_2'\right) x_3' + x_2 x_1' \left(x_2 + x_1' x_3\right)$$

Module II

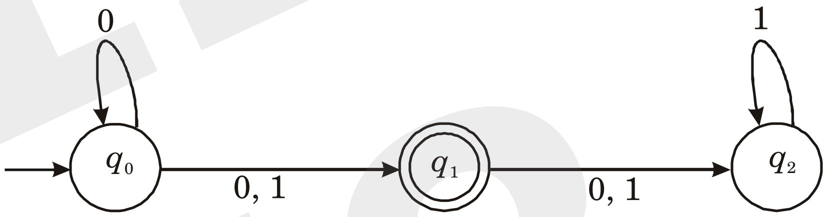
12. Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.

13. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

14. Prove that a simple graph is a tree if and only if any *two* distinct vertices are connected by a unique path.

Module III

15. Convert the nfa in the following figure into an equivalent deterministic machine.



16. Find a dfa that accepts all the strings on $\{0, 1\}$, except those containing the substring 001.

17. Show that the language

$$L\left\{awa : w \in \{a, b\}^*\right\}.$$

is regular.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Let $(X, +, \cdot, ')$ be a finite Boolean algebra. Then prove the following :

    (i) every non-zero element of X contains at least one atom.

    (ii) every two distinct atoms of X are mutually disjoint.

19. Prove that the set $Aut(G)$ of all automorphisms of a simple graph G is a group with respect to the composition $\circ$ of mappings as the group operation.

20. If the simple graphs G_1 and G_2 are isomorphic, then prove that $L(G_1)$ and $L(G_2)$ are isomorphic.

21. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite accepter, and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$, and $w \in \sum^+$, $\delta^*(q_i, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j.

(2 × 5 = 10 weightage)