D 114598 **(Pages : 2)** **Name.....**

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 *Short questions answerable within* 7.5 *minutes*
Answer **all** *questions, each question carries weightage* 1.

1. Write down the divergence of a vector V in orthogonal curvilinear co-ordinates.
2. Write down the orthogonality relation of a Hermite polynomials.
3. Define beta function.
4. What do you mean by pseudo tensors ? Give an example.
5. Define an orthogonal matrix with an example.
6. Write down the Rodrigues formula of Legendre polynomial and obtain $P_2(x)$ from Rodrigues formula.
7. Explain Schmidt orthogonalization.
8. Explain the convolution theorem with an example.

(8 × 1 = 8 weightage)

Section B

4 *essay questions answerable within* 30 *minutes.*
Answer any **two** *questions, each question carries weightage* 5.

9. Define a Fourier transform. Explain the properties of Fourier transforms.
10. Prove the orthogonality relation for the Bessel functions.

Turn over

11. Find the eigen values and eigen vectors of

$$H = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

12. Explain the Frobenius' method of finding solution to homogenous differential equation of second order by taking the example of linear oscillator.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions, each question carries weightage 3.*

13. Prove that every (second rank) tensor can be resolved into symmetric and antisymmetric parts.

14. Find the Fourier transform of a derivative.

15. Express the spherical polar unit vectors in terms of Cartesian unit vectors

16. Define the singular points of an ordinary differential equation, obtain the singular points of Bessel's differential equation

17. Prove that $\nabla \cdot r^n \hat{r} = (n+2) r^{n-1}$.

18. Two matrices A and B are each Hermitian. Find a necessary and sufficient condition for their product AB to be Hermitian.

19. Find the value of $\int_0^\infty e^{-x^4}\, dx$ in terms of gamma function.

(4 × 3 = 12 weightage)