D 122514 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2025

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Let $X = \{1, 2, 3\}$ and $\tau = \{X, \emptyset, \{1, 2\}, \{1, 3\}\}$. Verify whether τ is a topology on X.

2. Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \emptyset, \{1\}, \{1, 2\}\}$ be a topology on X. Find the closure of $A = \{1, 2\}$.

3. Let X, Y be topological spaces and X × Y be the product space and let A be open in X. Verify whether A × Y is open in X × Y.

4. Let $X = \mathbb{R}$ with indiscrete topology and $Y = \mathbb{R}$ with usual topology. Let $f : X \to Y$ be defined by $f(x) = x$ for all $x \in X$. Verify whether f is continuous.

5. Verify whether $\mathbb{R}$ with usual topology is connected.

6. Let $X = \mathbb{R} - \{0\}$ with usual topology. Find the connected components of X.

7. Let $X = \mathbb{R}$ with cofinite topology. Verify whether X is a T_1-space.

8. Let $\mathbb{R}$ be the real line and $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Find disjoint open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** *questions from each module.*
Each question has weightage 2.

Module I

9. Let $\mathcal{B}$ be a base for a topology τ on X and let $Y \subseteq X$. Show that

$$\mathcal{B}_Y = \{B \cap Y : B \in \mathcal{B}\}$$

is a base for the subspace topology on Y.

10. Let $f : X \to Y$ be a continuous function. Show that $f^{-1}(A)$ is closed in X for all closed sets A of Y.

11. Let X, Y be topological spaces and $f : X \to Y$ be a bijection which is continuous and open. Show that f is a homeomorphism.

Module II

12. Let $f_i : Y_i \to X$ be a family of functions from topological spaces Y_i to a set X. Show that

$$\mathcal{U} = \{A \subseteq X : f_i^{-1}(A) \text{ is open in } Y_i \text{ for all } i\}.$$

is the largest topology on X making each f_i continuous.

13. Let X be a first countable space and Y be any topological space. Let $f : X \to Y$ be such that if (x_n) is a sequence in X converging to x then $f(x_n)$ converges to $f(x)$. Show that f is continuous.

14. Let X be a connected space and $f : X \to Y$ be a continuous surjection. Show that Y is connected.

Module III

15. Let X be a finite T_1-space. Show that X is a discrete space.

16. Show that every metric space is a T_2-space.

17. Show that every T_4-space is a T_3-space.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Define second countable space and give an example.

    (b) Let X be a second countable space. Show that every open cover of X has a countable subcover.

19. (a) Define continuous function between two topological spaces.

    (b) Let X, Y be topological spaces and $f : X \to Y$. Show that the following are equivalent.

    i) If V is open in Y then $f^{-1}(V)$ is open in X.

    ii) If A is closed in Y then $f^{-1}(A)$ is closed in X.

    iii) For a subbase S of Y, $f^{-1}(V)$ is open in X for all $V \in \mathcal{S}$.

20. (a) Define connected space.

    (b) Show that a subspace A of the real line $\mathbb{R}$ is connected if and only if A is an interval.

21. (a) Show that every regular Lindeloff space is normal.

    (b) Show that every compact Hausdorff space is a T_4-space.

(2 × 5 = 10 weightage)