D 122516 **(Pages : 4)** **Name..................................**

Reg. No...............................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2025

(CBCSS)

Mathematics

MTH2C10—OPERATIONS RESEARCH

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that $f(x) = x^2, x \in \mathbb{R}$ ($\mathbb{R}$ is the set of real numbers), is a convex function.

2. Determine the maximum number of possible basic solutions to the problem :

   Maximize $2x_1 + x_2$

   subject to $x_1 - 3x_2 \leq 3$

   $2x_1 + x_2 \leq 20$

   $x_1 \leq 8$

   $x_1 \geq 0, x_2 \geq 0.$

3. What is meant by artificial variables in a linear programming problem ?

4. Prove that the dual of the dual is the primal.

5. What is meant by a balanced transportation problem ?

6. Define critical path associated with scheduling of sequential activities.

7. Describe the effect of deletion of variables on the optimal solution of an LP problem.

8. What is meant by the notion of dominance in game theory.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **six** *questions, by choosing two questions from each unit.*
Each question has weightage 2.

Unit I

9 Let, $K \subseteq E_n$ be a convex set $X \in K$, and $\int (X)$ a convex function. Prove that if $\int (X)$ has a relative minimum, it is also a global minimum.

10. Briefly describe the simplex method to solve a linear programming problem.

11. For the problem :

    Maximize $-6x_1 + 2x_2 - 4x_3 + 5x_1$

    subject to :

    $$4x_1 - x_2 + 2x_3 + 3x_4 \leq 1$$

    $$x_2 + 4x_3 - 2x_4 \leq 2$$

    $$x_1, x_2, x_3, x_4 \geq 0.$$

    determine the basic feasible solutions.

Unit II

12. In the optimal solutions of the primal and the dual, if a primal variable x_j is positive, then prove that the corresponding dual slack variable y_{m+j} is zero.

13. Write the dual of the LP problem :

    Maximize $2x_1 + 5x_2 + 3x_3$

    subject to :

    $$4x_1 + x_3 \leq 420$$

    $$2x_2 + 3x_3 \leq 460$$

    $$2x_1 + x_2 + x_3 \leq 500$$

    $$x_1, x_2, x_3 \geq 0.$$

    Without carrying out the simplex computation on either the primal or the dual problem, estimate a range for the optimal value of the objective function.

14. Solve the following transportation problem for minimum cost, starting with the degenerate solution $x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60$.

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

Unit III

15. Describe an algorithm to solve the problem of minimum path, when all arc lengths are non-negative.

16. Briefly describe the cutting plane method to solve an integer linear programming problem.

17. Solve the game with the pay-off matrix :

$$\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}.$$

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Prove that a vertex of the set S_F of feasible solutions is a basic feasible solution.

(b) Solve the following problem by big M method :

Maximize $4x_1 + 5x_2$

subject to : $2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \geq 1$

$x_1 + 4x_2 \geq 2$

$x_1 \geq 0, x_2 \geq 0.$

Turn over

19. (a) Prove that the value of the objective function f (X) for any feasible solution of the primal is not less than the value of the objective' function Φ (Y) for any feasible solution of the dual.

(b) Briefly describe the Caterer problem.

20. For the problem :

$$\text{Maximize } f = x_1 - x_2 + 2x_3$$

$$\text{subject to: } x_1 - x_2 + x_3 \leq 4$$

$$x_1 + x_2 - x_3 \leq 3$$

$$2x_1 - 2x_2 + 3x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0.$$

assuming x_4, x_5, x_6 respectively as the slack variables for the three constraints, the optimal table is the following :

Basis	*Values*	x_1	x_2	x_3	x_4	x_5	x_6
x_3	21	1		1		2	1
x_1	7	2			1	1	0
x_2	21	5	1			3	1
$-f$	18	2				1	1

Carry out sensitivity analysis when :

(i) A new constraint $2x_1 + x_2 + 2x_3 \leq 60$ is introduced.

(ii) Objective function changes to $3x_1 + x_2 + 5x_3$.

21. (a) What is meant by a zero-sum game.

(b) Write the LP problem corresponding to the rectangular game with the pay-off matrix :

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

Solve the game by solving the LP problem.

(2 × 5 = 10 weightage)