D 114583 **(Pages : 2)** **Name.....................................**

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Find the order of (2, 3) in the group $\mathbb{Z}_6 \times \mathbb{Z}_{12}$.

2. Find the maximum possible order for some element of $\mathbb{Z}_4 \times \mathbb{Z}_6$.

3. Let X be a G-set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exist $g \in G$ such that $gx_1 = x_2$. Prove that ~ is an equivalence relation on X.

4. Let $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_{12}$ be the homomorphism, where $\phi\ (1) = 10$. Find $Ker\ \phi$.

5. Show that $\mathbb{Z}$ has no composition series.

6. Find the number of Sylow 5-Subgroups of a group of order 15.

7. Give a presentation of $\mathbb{Z}_4$ involving two generators.

8. Find the multiplicative inverse of $i + 2j + 2k$ in the skew field of quaternions.

(8 × 1 = 8 weightage)

Section B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Let M be a maximal normal subgroup of a group G. Show that G/M is simple.

10. Describe all abelian groups up to isomorphism of order 360.

Turn over

11. Show that A_n is a normal subgroup of S_n and compute S_n/A_n.

MODULE II

12. Find all composition series of $S_3 \times \mathbb{Z}_2$.

13. Let G be a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$. Show that $G \simeq H \times K$.

14. Show that {2, 3} is a basis for $\mathbb{Z}_6$.

MODULE III

15. Find the sum and the product of the polynomials $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$.

16. Let F be a field and $f(x) \in F[x]$ be of degree 2 or 3. Show that $f(x)$ is irreducible if and only if $f(x)$ has no zero in F.

17. Prove that $\mathbb{Z}/5\mathbb{Z} \simeq \mathbb{Z}_5$.

(6 × 2 = 12 weightage)

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and isomorphic to $\mathbb{Z}_{mn}$ if and only if $\gcd(m, n) = 1$.

19. (a) State and prove Cauchy's Theorem.

    (b) Prove that no group of order 20 is simple.

20. (a) If N is a normal subgroup of a group G and H is any subgroup G, then prove that $H \vee N = HN = NH$.

    (b) Prove that every group is a homomorphic image of a free group.

21. (a) Prove that an element $a \in F$ is a zero of $f(x) \in F[x]$ if and only if $x - a$ is a factor of $f(x)$ in $F[x]$.

    (b) Prove that $25x^5 - 9x^4 - 3x^2 - 12$ is irreducible over $\mathbb{Q}$.

(2 × 5 = 10 weightage)