D 111188 **(Pages : 2)** **Name..................................**

Reg. No...............................

THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTH 3C 13—FUNCTIONAL ANALYSIS

(2019 Admission onwards)

Time : Theee Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Find the intersection of the unit ball in C[0, 1] with the subspace span{t}, where C[0, 1] denotes the set of all continuous functions on [0, 1] equipped with the supremum norm.
2. State and prove the Pythagorean theorem.
3. State the Holder's inequality.
4. When do you say that two different norms defined on a space are equivalent ?
5. Define a separable normed space and give an example for a separable space.
6. When do you say that a space is reflexive ? Give an example for a space which is not reflexive.
7. Give an example to show that weak convergence does not imply strong convergence.
8. State the Banach open map theorem.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

Unit I

9. Show that the sequence space $l_p, 1 \leq p \leq \infty$ is a complete normed space.
10. Prove that if a normed linear space X contains linearly independent vectors x and y such that $\|x\| = \|y\| = 1, \|x+y\| = \|x\| + \|y\|$, then there exists a line segment contained in the unit sphere of X.
11. For every sequence of scalars $a = (a_i)$ and $b = (b_i)$ and for $1 \leq p \leq \infty$ prove that $\|a+b\|_p \leq \|a\|_p + \|b\|_p$.

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Unit II

12. State and prove the Parseval's identity.
13. Prove that every seperable Hilbert space has an orthonormal basis.
14. Prove that if a Hilbert space contains an uncountable orthonormal system, it cannot be separable.

Unit III

15. Prove that if E is a finite dimensional subspace of a normed space X, then E is a closed subspace.
16. If A and B are invertible operators, prove that AB is also invertible and its inverse satisfies $(AB)^{-1} = B^{-1} A^{-1}$.
17. Prove that the set $K(X \mapsto Y)$ of compact operators from X to Y is a closed subspace of $L(X \mapsto Y)$, the linear space of bounded operators from X to Y.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. Prove that l^{∞}, the set of bounded sequences is a Banach space.
19. When do you say that a normed space is separable ? Show that the Hilbert space H is separable if and only if there exists a complete orthonormal system $\{e_i\}_{i \geq 1}$.
20. Prove that for $1 < p < \infty, 1/p + 1/q = 1, (l_p)^* = l_q$, where $(l_p)^*$ denotes the dual space of l_p, the space of all sequences (x_n) with $\sum |x_n|^p < \infty$.
21. Let X be a normed space and Y be a complete normed space. Prove that $L(X \mapsto Y)$, the linear space of bounded operators from X to Y is a Banach space.

(2 × 5 = 10 weightage)

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