D 111189 **(Pages : 3)** **Name.................................**

Reg. No...............................

THIRD SEMESTER M.Sc. (CBCSS) (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Solve $u_x = 1$ subject to the condition $u(0, y) = g(y)$.
2. For the equation $u_{xx} + 4u_{xy}\, u_x = 00$ find a canonical transformation $q = q(x, y)$, $r = r(x, y)$ and the corresponding canonical form.
3. Describe Domain of dependence and region of influence.
4. Show that the only possible value for the eigen value problem

   $\dfrac{d^2X}{dx^2} + \lambda X = 0,\ 0 < x < L, X(0) = X(L) = 0$ .are positive real numbers.
5. Show that if u be a function in $C^2(D)$ satisfying the mean value property at every point in D then u is harmonic in D.
6. Prove that every harmonic function in D are infinitely differentiable on D.
7. State four properties of Green's function.
8. Find the resolvent kernel of the Volterra integral equations with the kernel $K(x, \xi) = 1$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit.*
Each question has weightage 2.

Unit 1

9. Show that the Cauchy problem $u_x + u_y = 1, u(x \cdot x) = x$ has uniquely many solutions.
10. Solve the equation $u_x + u_y + u = 1$ subject to the initial condition $u\left(x, x + x^2\right) = \sin(x), x > 0$.
11. Prove that type of the equation is an intrinsic property of the equation and is independent of the particular co-ordinate system.

Turn over

UNIT 2

12. Solve $u_t - 17u_{xx} = 0, 0 < x < \pi, t > 0$

$$u(0,t) = u(\pi,t) = 0, t \geq 0$$
$$u(x,0) = f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{\pi}{2} \\ 2, & \frac{\pi}{2}, \leq x \leq \pi. \end{cases}$$

13. Show that Laplace equation over the plane is the solution that is symmetric about the origin and find its fundamental solution.

14. Find the harmonic function in the unit square satisfying the Dirichlet conditions $u(x,0) = 1 + \sin(\pi x), u(x,1) = 2, u(0,y) = u(1,y) = 1 + y.$

UNIT 3

15. Formulate the integral equation corresponding to the differential equation $y'' + xy = 1, y(0) = y(1) = 0.$

16. Find the eigenvalues and eigenfunctions of the homogeneous integral equations :

$$y(x) = \lambda \int_0^1 e^{x+\xi} y(\xi)\, d\xi.$$

17. Show that the eigenfunctions of a symmetric kernel, corresponding to different eigenvalue are orthogonal.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. Use the method of characteristic strips to solve the non-linear eikonal equation $p^2 + q^2 = n^2$ subject to the condition $u(x,1) = n\sqrt{1+x^2}$, where n is a constant parameter.

19. For the problem $u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0$ with initial conditions :

$$u(x,0) = f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$
$$u_t(x,0) = \begin{cases} 4, & 1 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $u(x, 1)$.

(b) Find $\lim_{t\to\infty} u(5,t)$.

(c) Find the set of all points where the solution is singular.

(d) Find the set of all points where the solution is continuous.

20. Apply the method of separation of variables to solve the problem of a vibrating string without external forces and with two clamped but free ends.

21. Determine the resolvent kernal of $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\, d\xi$ where $k(x,\xi) = 1 - 3x\xi$, for what value of λ the solution does not exists. Obtain the solution of the above integral equation.

(2 × 5 = 10 weightage)