D 111193 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER M.Sc. (CBCSS) (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Every r.v. has a unique distribution function. Is the converse true ? Justify.
2. Let X be a discrete r.v. with pmf $P\left[X = \frac{(-1)^{j+1}3^j}{j} = \frac{2}{3^j};\ j = 1, 2, \ldots\right]$. Find E(X) if it exists.
3. State the Markov inequality and deduce the Chebyshev's inequality from it.
4. Define a mixture distribution. If $F_X(x) = 1 - p + p\left(1 - e^{-\theta x}\right)$, identify the discrete part and continuous part of the decomposition.
5. Define independence of random variables.
6. If $X_1, X_2, \ldots X_n$ are iid random variables having exponential distribution with parameter θ, obtain the distribution of $X_{(1)} = \min i\{X_1, X_2, \ldots, X_n\}$.
7. If $X_n \xrightarrow{r} X$, then prove that $X_n \xrightarrow{P} X$. Is the converse true always. Establish.
8. Check whether WLLN holds for the sequence $\{X_n, n \geq\}$ of random variables defined as $P\left(X_n = \sqrt{n}\right) = \frac{1}{2} = P\left(X_n = -\sqrt{n}\right)$.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

Unit 1

9. For an integer valued r.v. X, with $P(X = n) = p_n$ and $P(X \leq n) = q_n$ so that $\sum_{i=0}^{n} p_i = q_n$, then prove $\sum_{n=0}^{\infty} P(X \leq n)s^n = \frac{P_X(s)}{1-s}, |s| \leq 1$, where $P_X(s)$ is the probability generating function of X.

Turn over

10. Obtain the distribution of X^c if X has a Weibull distribution with $F(x) = 1 - e^{-x^c}, x > 0, c > 0$.

11. Show that $X_n \xrightarrow{r} X \Rightarrow E|X_n|^r \to E|X|^r$.

UNIT 2

12. Define the joint central moments (first four) for a bivariate distribution.

13. If $f(x, y) = \frac{1}{4}(1 + xy), |x| < 1, |y| < 1,$ then show that X^2 and Y^2 are independent but X and Y are not.

14. Prove that Var (X) = E[Var(X | Y)] + V[E(X | Y)].

UNIT 3

15. Prove that $X_n \xrightarrow{P} 0 \leftrightarrow E\left[\frac{|X_n|}{1+|X_n|}\right] \to 0.$

16. Examine whether WLLN holds for the sequence $\{X_n, n \geq\}$ of i.i.d. random variables defined as $P(X_n = \pm 2^n) = 2^{-(2n+1)}; n \geq 1$ and $P(X_n = 0) = 1 - 2^{-2n}$.

17. State and prove the Linberg-Levy's form of Central Limit Theorem.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. Prove that a distribution function is symmetric if and only if its characteristic function is real and even.

19. The joint pdf of two random variables X and Y is

$$f(x, y) = \begin{cases} 8xy, & 0 \leq x < y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find (i) the marginal pdfs of X and Y ; (ii) the conditional pds ; (iii) Var(X) and Var(Y) ; (iv) E(Y/X = x), Var(Y/X = x) ; and (v) E(XY/Y = x).

20. Let X_1 follows Gamma (α, β_1) and X_2 follows Gamma (α, β_2) and are independent with pdfs $f(x_1) = \frac{\alpha^{\beta_1} e^{-\alpha x_1} x_1^{\beta_1 - 1}}{\Gamma(\beta_1)}$, for $x_1 \geq 0$ and $f(x_2) = \frac{\alpha^{\beta_2} e^{-\alpha x_2} x_2^{\beta_2 - 1}}{\Gamma(\beta_1)}$, for $x_2 \geq 0$, then what are the distributions of $X_1 + X_2$ and the distribution of $\frac{X_1}{X_2}$? Are they independent ?

21. (a) Show that every sequence of independent random variables with uniformly bounded variance obeys the SLLN.

(b) Prove that the WLLN holds for the sequence of random variables $\{X_n, n \geq 1\}$ defined as $P\left(X_n = \pm n^{\alpha}\right) = \frac{1}{2}$ if and only if $< \frac{1}{2}$.

(2 × 5 = 10 weightage)

D 111193-A **(Pages : 5)** **Name..................................**

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THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

[Improvement Candidates need not appear for MCQ part]

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 3E 04—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. If X and Y are two random variables with means $\overline{X}$ and $\overline{Y}$ respectively, then the expression $E\left[\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)\right]$ is called :

   (A) Variance of X. (B) Variance of Y.

   (C) Cov (X, Y). (D) Moments of X and Y.

2. The random variables X and Y have variances 0.2 and 0.5 respectively. Let Z = 5X – 2Y. The variance of Z is ?

   (A) 3. (B) 7.

   (C) 4. (D) 5.

3. The weight of persons in a country is a r.v. of the type :

   (A) Continuous r.v.

   (B) Discrete r.v.

   (C) Neither discrete nor continuous.

   (D) Discrete as well as continuous.

4. Consider a r.v. X that takes values + 1 and – 1 with probability 0.5 each. The value of the distribution function F(x) = at x = – 1 and + 1 are :

   (A) 0 and 0.5. (B) 0 and 1.

   (C) 0.5 and 1. (D) 0.25 and 0.75.

5. Two random variables X and Y are said to be independent if :

   (A) E(XY) = 1. (B) E(XY) = 0.

   (C) E(XY) = E(X) E(Y). (D) E(XY) = any constant value.

6. A discrete r.v. has probability mass function $p(x) = kq^x p,\ p+q=1,\ x=2,3,4,\ldots$ the value of k should be equal to :

   (A) $1/q^2$. (B) $1/p$.

   (C) $1/q$. (D) $1/pq$.

7. If X and Y two independent variables and their expected values are $\bar{X}$ and $\bar{Y}$ respectively then :

   (A) $E\{(X-\bar{X})(Y-\bar{Y})\}=0.$

   (B) $E\{(X-\bar{X})(Y-\bar{Y})\}=1.$

   (C) $E\{(X-\bar{X})(Y-\bar{Y})\}=C$ (constant).

   (D) All the above.

8. If X is a random variable and its p.d.f. if $f(x)$, E(log x) represents :

   (A) Arithmetic mean.
   (B) Geometric mean.
   (C) Harmonic mean.
   (D) Logarithmic mean.

9. If X and Y are two random variables, the covariance between the variables $aX + b$ and $cY + d$ in terms of COV (X, Y) is :

   (A) COV $(aX + b, cY + d)$ = COV (X, Y).
   (B) COV $(aX + b, cY + d) = abcd \times$ COV (X, Y).
   (C) COV $(aX + b, cY + d) = ac$ COV (X, Y) + bd.
   (D) COV $(aX + b, cY + d) = ac$ COV (X, Y).

10. The equation $|\rho| = 1$ holds if and only if there exist constants $a \neq 0$ and b such that ————.

    (A) $P\{aX+b=1\}=1.$
    (B) $P\{aX+b=1\}=0.$
    (C) $P\{aX+b=0\}=1.$
    (D) $P\{aX=1\}=1.$

11. If $V(x) = 1$, then $V(2x \pm 3)$ is :

    (A) 5.
    (B) 13.
    (C) 4.
    (D) 10.

12. The correlation coefficient ρ between two r.vs satisfies :

    (A) $|\rho| = 1.$
    (B) $|\rho| \leq 1.$
    (C) $|\rho| < 0.$
    (D) $\rho > 1.$

13. Let (X, Y) be an RV of the discrete type, then the joint probability mass function of (X, Y) defined as :

(A) $p_{ij} = P\{X = x_i, Y = y_j\}, i = 1, 2, \ldots j = 1, 2, \ldots.$

(B) $p_{ij} = P\{X > x_i, Y > y_j\}, i = 1, 2, \ldots j = 1, 2, \ldots.$

(C) $p_{ij} = P\{X < x_i, Y < y_j\}, i = 1, 2, \ldots j = 1, 2, \ldots.$

(D) $p_{ij} = P\{X = x_i, Y > y_j\}, i = 1, 2, \ldots j = 1, 2, \ldots.$

14. If (X, Y) is an RV of the continuous type, then marginal pdf of X is ————.

(A) $f_1(x) = \int_{-\infty}^{\infty} f(x, y)\, dy.$

(B) $f_1(x) = \int_{-\infty}^{0} f(x, y)\, dy.$

(C) $f_1(x) = \int_{0}^{\infty} f(x, y)\, dy.$

(D) None of the above.

15. Let (X, Y) be an RV of the continuous type, then conditional PDF of X, given Y = y defined as :

(A) $f_{X/Y}(x \mid y) = \dfrac{f(x, y)}{f_2(y)}.$

(B) $f_{Y/X}(y \mid x) = \dfrac{f(x, y)}{f_2(y)}.$

(C) $f_{X/Y}(x \mid y) = \dfrac{f(x, y)}{f_1(x)}.$

(D) $f_{Y/X}(y \mid x) = \dfrac{f(x, y)}{f_1(x)}.$

16. Two RVs X and Y, of the continuous type are independent if and only if :

(A) $f(x, y) = f_1(x) + f_2(y).$

(B) $f(x, y) = f_1(x) - f_2(y).$

(C) $f(x, y) = f_1(x) / f_2(y).$

(D) $f(x, y) = f_1(x) f_2(y).$

17. If $f(x)$ is a probability density function of a continuous random variable then ————.

(A) $\int_{-\infty}^{\infty} f(x)\, dx = 0.$

(B) $\int_{-\infty}^{\infty} f(x)\, dx = 1.$

(C) $\int_{-\infty}^{\infty} f(x)\, dx > 1.$

(D) $\int_{-\infty}^{\infty} f(x)\, dx < 0.$

18. The expected value of the constant b is ————.

    (A) 0. (B) 1.

    (C) $1/b$. (D) b.

19. If you roll two dice, what is the likelihood that you will roll two numbers that are the same ?

    (A) 1/6. (B) 2/36.

    (C) 1/36. (D) 1/12.

20. Suppose $X \sim P(\lambda)$. What is the distribution of $Y = \frac{(X - \lambda)}{\sqrt{\lambda}}$?

    (A) $N(\lambda, \sigma)$. (B) $N(0, 1)$.

    (C) $N(\mu, 1)$. (D) $N(\lambda, 1)$.