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D 121290 **(Pages : 3)** **Name..............................**

Reg. No...............................

FOURTH SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2025

Mathematics

MTH4E12—REPRESENTATION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

*Answer **all** questions.*
Each question carries a weightage 1.

1. Let ϕ be the character of a representation of a group G given by $A(x) = I_3$ for each $x \in G$. Find $\phi(x)$.
2. Let $G = \{1, a : a^2 = 1\}$ be the cyclic group of order 2 and $\mathbb{R}^2$ be a G-module given by $(x, y)a = (y, x)$. Verify whether $U = \{(x, 0) : x \in \mathbb{R}\}$ is a submodule.
3. Let υ be the natural character of the symmetric group S_3. Find $v(a)$ where $a = (12)$.
4. Let ξ be the alternating character of the symmetric group S_3. Find $\langle \xi, \xi \rangle$.
5. Let ρ be the the character of the right regular representation of the cyclic group $G = \{1, a, a^2 : a^3 = 1\}$. Find $\rho(1)$.
6. Show that every simple character of an abelian group is a linear character.
7. Let G be the symmetric group S_3 and H be the subgroup A_3. Let ϕ be the trivial character of H Find $\phi^G(a)$ where $a \in G$.
8. Describe the degrees of the simple characters of the symmetric group S_3.

(8 × 1 = 8 weightage)

Part B

*Answer any **two** questions from each module.*
Each question carries a weightage 2.

Module I

9. Let $G = \{1, a, a^2 : a^3 = 1\}$ be the cyclic group of order 3. Describe a G-module structure on $\mathbb{R}^3$.

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10. Let V be a G-module and U be a submodule of V. Show that V/U is also a G-module.

11. Let V be a G-module corresponding to the permutation representation of the symmetric group S_3. Let $\{v_1, v_2, v_3\}$ be a basis of V. Show that the subspace U generated by $v_1 + v_2 + v_3$ is a G-submodule.

Module II

12. Show that the number of simple characters of a finite group G is less than or equal to the order of the group.

13. Find the number of simple character of the alternating group A_4.

14. Describe the character table of the cyclic group of order 3.

Module III

15. Let v be the natural character of S_3. Show that $\phi(x) = v(x) - 1$ is a character of S_3.

16. Let G be a transitive permutation group. Describe the stabilizer of a symbol α. Show that the stabilizer of α is a subgroup of G.

17. Define doubly transitive permutation group and give an example.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

18. (a) Define G-homomorphism between G-modules.

(b) Let $\theta : V \to U$ be a G-homomorphism between G-modules V and U Show that

(i) ker θ is a G-submodule of V.

(ii) Im θ is a G-submodule of U.

(iii) If V and U are irreducible then θ is either the zero map or an isomorphism.

19. (a) Define commutant algebra $\mathcal{C}(A)$ of a representation $A(x)$.

(b) Let $A(x)$ be a representation of a finite group over an algebraically closed field K and let

$$A(x) \sim diag\left(A_1(x), A_2(x), \ldots, A_k(x)\right)$$

where $A_i(x)$ are inequivalent irreducible representation of G. Show that dimension of $\mathcal{C}(A)$ is k.

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20. (a) Let $A(x)$ be a representation of a group G wth character ϕ. Show that

    (i) $M = \{u \in G : \phi(u) = \phi(1)\}$ is a normal subgroup of G.

    (ii) For each $Mx \in G/M$ define $A_u(Mx) = A(x)$. Then A_0 is a representation of G/M.

    (b) Describe the character table of A_4.

21. (a) State Frobenius Reciprocity theorem.

    (b) With the usual notations prove that $\langle \psi^G, \phi \rangle_G = \langle \psi, \phi_H \rangle_H$ where H is a subgroup of G.

    (c) Describe the character table of the Alternating group A_5.

(2 × 5 = 10 weightage)

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D 121290-A **(Pages : 5)** **Name..................................**

Reg. No..............................

FOURTH SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2025

Mathematics

MTH 4E 12—REPRESENTATION THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MTH 4E 12—REPRESENTATION THEORY

(Improvement candidates need not appear for MCQ part)

1. A homomorphism θ from an abstract group to a concrete group is known as :

   (A) An isomorphism. (B) An epimorphism.

   (C) A representation. (D) Endomorphism.

2. Let $A(x)$ be a faithful representation of a group G. Which of the following is false ?

   (A) $A(x)\, A(y) = A(xy)$ for every $x, y \in G$.

   (B) $A(x) = I$ if and only if $x = e$ in G.

   (C) $A(x) = x$ for all $x \in G$.

   (D) Ker(A) is a normal subgroup of G.

3. Let $A(x)$ be a matrix representation of a group G and let $\phi(x)$ be the character of $A(x)$. Which of the following is true ?

   (A) Equivalent representation have the same character.

   (B) Equivalent representation need not have the same character.

   (C) There exists $x, y \in G$ with x and y conjugate in G and $\phi(x) \neq \phi(y)$.

   (D) None of these.

4. Let $V = [e_1, e_2, e_3]$ be a basis of $\mathbb{R}^3$ and let A denote the matrix representation of the linear map α on $\mathbb{R}^3$ defined by $\alpha(x, y, z) = (y, x + y, z)$. Then, A is :

   (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. (B) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. (D) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

5. If $\phi(x)$ and $\phi^{\dagger}(x)$ are the characters of $A(x)$ and $A^{\dagger}(x)$ respectively, then

   (A) $\phi^{\dagger}(x) = \phi(x)$. (B) $\phi^{\dagger}(x) = \overline{\phi(x)}$.

   (C) $\phi^{\dagger}(x) = \overline{\phi(x^{-1})}$. (D) $\phi^{\dagger}(x) = \phi(x^{-1})$.

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6. Let $G = \{\ldots, x^{-2}, x^{-1}, x^{0} = 1, x^{1}, x^{2}, \ldots$ and $A(x^{h}) = \begin{bmatrix} 1 & 0 \\ h & 1 \end{bmatrix}$. Then :

   (A) A(x) is diagonalizable.

   (B) A(x) is completely reducible.

   (C) A(x) is not a representation of G.

   (D) A(x) is lower triangular but not diagonalizable.

7. Suppose that U and V are irreducible G-modules over K. Then a G-homomorphism $\theta : V \to U$ is :

   (A) Always an isomorphism.

   (B) Always the zero map.

   (C) Neither zero nor an isomorphism.

   (D) Either zero or an isomorphism.

8. The character of an irreducible representation is called :

   (A) Trivial character.

   (B) Compound character.

   (C) Simple character.

   (D) None of these.

9. Let $\chi^{(1)}, \chi^{(2)}, \ldots \chi^{(s)}$ be distinct simple characters of a group of order g. Then,

   (A) s is always equal to g.

   (B) $s \leq g$.

   (C) $s \geq g$.

   (D) s is always different from g.

10. Let D be a representation of the group S_3 of order 2 and 1 be the identity in S_3. Then the character of D(1) is :

   (A) 1.

   (B) 2.

   (C) 0.

   (D) – 1.

11. Let R(x) = $r_{ij}(x)$ be the right regular representation of a G-module $G_{\mathbb{C}}$ with basis vectors $[x_1 (=1), x_2, \ldots, x_g]$ and let $\rho(x)$ be the character of R(x). Then, ρ is the g-tuple :

   (A) (g, 1, 1, 1, …, 1).

   (B) (0, 0, 0,0, …, g).

   (C) (1, 1, 1, …, 1, g).

   (D) (g, 0, 0, 0, …, 0).

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12. Let G be a group of order g and let the number of elements in a conjugacy class C_α, $(1 \le \alpha \le k)$ be denoted by h_α. Then, the equation $\sum_{\alpha=1}^{k} h_\alpha = g$ is called :

    (A) Character relation of first kind.

    (B) Class equation.

    (C) Character relation of second kind.

    (D) None of these.

13. Let G be an abelian group of order g. Then for $x \in G$, the conjugacy class of x has :

    (A) $g - 1$.
    (B) At least two elements.
    (C) Precisely one element.
    (D) g elements.

14. Let A be a periodic matrix over $\mathbb{C}$. Then :

    (A) A is diagonalizable.
    (B) A is not diagonalizable.
    (C) A is nilpotent.
    (D) A = I.

15. Let $[1, 1, 1, 1, 2]^T$ and $[1, 1, 1, 1, x]^T$ be two column vectors of character table of D_4. Then, x is :

    (A) – 1.
    (B) – 2.
    (C) 0.
    (D) 1.

16. Which of the following is in A_6 ?

    (A) (123)(56).
    (B) (1256).
    (C) (56).
    (D) (1234)(56).

17. The set of all elements of G that commute with x is called the :

    (A) Centraliser of x.
    (B) Commutator subgroup generated by x.
    (C) Stabilizer of x.
    (D) Normalizer of x.

18. Let χ denote the permutation character of A_5. Then $\chi((123))$ is :

    (A) – 1.
    (B) 2.
    (C) 3.
    (D) 1.

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19. Which of the following is true ?

(i) A : 1, (12)(34), (13)(24), (14)(23) is transitive.

(ii) B : 1, (12), (34), (12) (34) is transitive.

(A) Both (i) and (ii).
(B) (ii) but not (i).
(C) Only (i).
(D) Neither (i) nor (ii).

20. Let G be a doubly transitive group with natural character $v(x)$, then the function $v(x) - 1$ is :

(A) Always compound.
(B) Always simple.
(C) Sometimes compound.
(D) None of these.

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