D 121289 **(Pages : 2)** **Name..................................**

Reg. No..............................

FOURTH SEMESTER M.Sc. (CBCSS) (REGULAR/SUPPLEMENTARY) DEGREE EXAMINATION, APRIL 2025

Mathematics

MTH4E11—GRAPH THEORY

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer **all** *questions.*
Each questions carries a weightage 1.

1. Define a tree. Prove that a graph G is a tree if and only if every two points are joined by a unique path.
2. Define vertex cut set and edge cut set of a simple graph G. Illustrate with an example.
3. Prove that in any graph the number of vertices of odd degree is even.
4. Define chromatic number $\chi(G)$ of a graph G. Prove that $\chi(K_n) = n$.
5. Define radius of a connected graph. Illustrate with an example.
6. Prove that every critical graph is a block.
7. Prove that in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.
8. Prove that in a k-critical graph $(k \geq 2)$, no vertex cut is a clique.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage of 2.

Module I

9. Prove that every connected graph contains a spanning tree.
10. Find the number of spanning trees in $K_{3,3}$.
11. Prove that a subset S of V is independent if and only if V\S is a covering of G.

Module II

12. If G is a k-regular bipartite graph with $k > 0$, then prove that G has a perfect matching.

Turn over

13. Prove that for any graph G of order $n, \alpha' + \beta' = n$, where α' is the edge independence number of β' is the edge covering number of G.
14. Prove that every tournament has a directed Hamilton path.

Module III

15. Prove that for any integer k, there exists k-chromatic graph containing no triangle.
16. Prove that $K_{3,3}$ is non-planar.
17. Prove that all planar embeddings of a given connected planar have the same number of faces.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Prove that a non-empty connected graph is Euler trail if and only if it has at most two vertices of odd degree.
19. If G is a simple (p, q) graph with $p \geq 3$ and $\delta \geq \frac{p}{2}$, then prove that G is Hamiltonian.
20. If the graph G is 4-chromatic, then prove that G contains a subdivision of K_4.
21. Prove that every planar graph is 5-vertex colourable.

(2 × 5 = 10 weightage)