D 131316 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define Lattice.
2. Define total order.
3. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that for all elements x, y and z of X,

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z.$$

4. Define labeled graph.
5. Prove that the Petersen graph P is nonplanar.
6. Show that every plane triangulation of order $n \geq 4$ is 3-connected.
7. Define language accepted by an nfa. Give an example.
8. Define dfa.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Define a chain in a poset. Prove that the intersection of two chains is a chain.

10. State the law of double complementation in a Boolean algebra.

11. Write the following Boolean function in their disjunctive normal form.

$$f\left(x_1, x_2, x_3\right) = \left(x_1 + x_2'\right) x_3' + x_2 x_1' \left(x_2 + x_1' x_3\right).$$

MODULE II

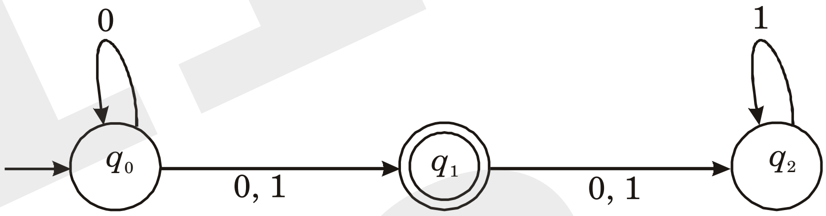
12. In any group of n persons $(n \geq 2)$, prove that there are at least two with the same number of friends.

13. Prove that an edge $e = xy$ is a cut edge of a connected graph G if and only if there exist vertices u and v such that e belongs to every u-v path in G.

14. Prove that every connected graph contains a spanning tree.

MODULE III

15. Convert the nfa in the following figure into an equivalent deterministic machine.



16. Find a deterministic finite accepter that recognizes the set of all strings on $\sum = \{a, b\}$ starting with the prefix ab.

17. Show that the language

$$L = \left\{awa : w \in \{a, b\}^*\right\}$$

is regular.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Let X be a finite set and $\leq$ a partial order on X. Define a binary relation R on X by $x\mathrm{R}y$ if and only if y covers x (w.r.t. $\leq$). Then prove that $\leq$ is generated by R.

19. Prove that the number of edges in a tree on n vertices is $n - 1$. Also prove that a connected graph on n vertices and $n - 1$ edges is a tree.

20. State and prove Euler Formula.

21. Let $\mathrm{M} = (\mathrm{Q}, \Sigma, \delta, q_0, \mathrm{F})$ be a deterministic finite accepter, and let $\mathrm{G_M}$ be its associated transition graph. Then prove that for every $q_i, q_j \in \mathrm{Q}$, and $\mathrm{w} \in \sum^{+}$, $\delta^{*}(q_i, w) = q_j$ if and only if there is in $\mathrm{G_M}$ a walk with label w from q_i to q_j.

(2 × 5 = 10 weightage)