D 131317 **(Pages : 3)** **Name....................................**

Reg. No................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 1C05—NUMBER THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*

Each question carries a weightage 1.

1. Show that the Mobius function $\mu(n)$ is completely multiplicative.
2. If f is multiplicative then show that $f(1) = 1$.
3. State and prove Legendre's identity.
4. For $x \geq 1$, show that $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$.
5. For $x > 0$, show that $0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}$.
6. Explain enciphering and deciphering transformation.
7. State Reciprocity law for Legendre's symbol.
8. Define the Jacobi symbol and evaluate the Jacobi symbol $(2 \mid P)$, where P is an odd positive integer.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*

Each question carries a weightage 2.

Module I

9. For $n \geq 1$, show that $\sum_{d|n} \mu(d) = \left[\frac{1}{n}\right]$.

10. Let f be a multiplicative function. Prove that f is completely multiplicative if and only if $f(p^a) = f(p)^a$ for all primes p and for all integers $a \geq 1$.

11. For $x \geq 2$ prove that $\sum_{p \leq n} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$, where the sum is extended over all primes $\leq x$.

Module II

12. For all $x \geq 1$, show that $\sum_{n \leq x} \frac{\Lambda(n)}{n} = \log x + O(1)$.

13. For $x \geq 2$, prove that $\vartheta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$ and $\pi(x) = \frac{\vartheta(x)}{\log x} + \int_2^x \frac{\vartheta(t)}{t \log^2 t} dt$.

14. State and prove Abel's identity.

Module III

15. Determine those odd primes for which 3 is a quadratic residue and those for which it is a quadratic non-residue.

16. State and prove Quadratic Reciprocity Law for Jacobi symbol.

17. How do classical and public Key cryptosystem differ ?

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*

Each question carries a weightage 5.

18. Given f with $f(1) = 1$. Then prove that :

    (a) f is multiplicative if and only if $f\left(p_1^{a_1} \ldots p_r^{a_r}\right) = f\left(p_1^{a_1}\right) \ldots f\left(p_r^{a_r}\right)$ for all primes p_t and all integers $a_i \geq 1$.

    (b) If f is multiplicative, then f is completely multiplicative if and only if $f\left(p^a\right) = f\left(p^a\right)$ for all primes p and all integers $a_i \geq 1$.

19. State and prove Euler's summation formula.

20. Show that the following relations are logically equivalent :

    (a) $\lim\limits_{x \to \infty} \dfrac{\pi(x) \log(x)}{x} = 1.$

    (b) $\lim\limits_{x \to \infty} \dfrac{\vartheta(x)}{x} = 1.$

    (c) $\lim\limits_{x \to \infty} \dfrac{\psi(x)}{x} = 1.$

21. State and prove Gauss's lemma.

(2 × 5 = 10 weightage)