D 132450 **(Pages : 3)** **Name.....**

Reg. No.....

FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2025

Mathematics

MAT1CJ101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours Maximum : 70 Marks

Section A

All questions can be answered.

Each question carries 3 *marks.*

(*Ceiling* 24 *marks*).

1. Draw the graph of the equation $\frac{x^2}{4}+\frac{y^2}{9}=1$.

2. Find the domain of the function $f(x)=\frac{1}{x(x-1)(x-2)}$ in real numbers.

3. Find $\lim_{x\to 0^+} \ln \frac{2}{x^2}$.

4. Discuss the continuity of the function $\sqrt{2x+3}$.

5. Find $\frac{dw}{dt}$ if $w = \tan x$ and $x = 4t^3 + t$.

6. If $y = \sin(3x)$, then find $\frac{d^2y}{dx^2}$.

7. Find the absolute maximum and minimum values of $f(x)=(x-2)^3$ on the closed interval [1, 4].

Turn over

8. Verify that the hypotheses of the Mean-Value Theorem for the function $f(x) = x^2 - x$ on the interval $[-3, 5]$ and find all values of c in that interval that satisfy the conclusion of the theorem.

9. Find the relative extrema of the function $f(x) = \dfrac{x+1}{x-1}$.

10. Give a graph of the function $f(x) = x + \sin x$ and identify the locations of all relative extrema and inflection points.

Section B

All questions can be answered.

Each question carries 6 *marks.*

(*Ceiling* 36 *marks*)

11. If $\lim\limits_{x \to 2} \dfrac{f(x) - 5}{x - 2} = 3$, then find $\lim\limits_{x \to 2} f(x)$.

12. Find $\lim\limits_{x \to 4} \dfrac{3 - x}{x^2 - 2x - 8}$.

13. If functions $f(x)$ and $g(x)$ are continuous for $0 \le x \le 1$, could $f(x)/\, g(x)$ possibly be discontinuous at a point of $[0, 1]$? Give reasons for your answer.

14. Use implicit differentiation to find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$, if $\sqrt{y} = 3x^2 + \tan x$.

15. Find the absolute extrema values of $F(t) = 8t - t^4$ on $[-2, 1]$.

16. Find the critical points of $f(x) = x^{1/3}(x - 4)$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

17. Find the asymptotes of the curve $f(x) = 2 + \frac{\sin x}{x}$.

18. Solve $\lim_{x \to -\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$.

Section C

Answer any **one** *questions.*

Each question carries 10 *marks.*

19. At time $t \geq 0$, the velocity of a body moving along the s–axis is $v = t^2 - 4t + 3$.

    (a) Find the body's acceleration each time the velocity is zero.

    (b) When is the body moving forward ? moving backward ?

    (c) When is the body's velocity increasing ? decreasing ?

20. (a) Show that $f(x) = x^3 + \frac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

    (b) Suppose the derivative of the function $y = f(x)$ is $y' = (x-1)^2 (x-2)(x-4)$. At what points, if any, does the graph of f have a local minimum, local maximum, or point of inflection ?

(1 × 10 = 10 marks)