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## FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(CUFYUGP)

B. Sc. Mathematics Honours

MAT1MN105-MATRIX THEORY

2024 Admission onwards

Maximum Time :2 Hours	Maximum Marks :70
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## Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	Solve $\begin{aligned}x + 2y &= 3 \\ 3x + y &= 1\end{aligned}$
2	Find all values of $k$ for which the augmented matrix $\left[ \begin{array}{ccc} 1 & k & -1 \\ 4 & 8 & -4 \end{array} \right]$ corresponds to a consistent linear system
3	Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 6 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 7 & 1 \\ 8 & 3 & 9 \end{bmatrix}$ . Show that $tr(A + B) = tr(A) + tr(B)$
4	Let $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 6 \end{bmatrix}$ . Show that $(AB)^{-1} = B^{-1}A^{-1}$
5	Compute $p(A)$ for the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ and the polynomial $x^2 + 3x - 2$ .
6	Use the arrow technique to evaluate the determinant $\begin{vmatrix} 2 & 6 & 5 \\ 9 & 6 & 10 \\ 11 & 2 & 1 \end{vmatrix}$
7	Find the determinant of $\begin{bmatrix} 0 & 0 & 100 & 1002 \\ 4 & 5 & 1000 & 2000 \\ 0 & 0 & 0 & 10 \\ 0 & 3 & 25 & 300 \end{bmatrix}$
8	Let $P$ be the point $(1, 7, -3)$ and $Q$ the point $(4, -7, 3)$ . Find the point on the line segment connecting the points $P$ and $Q$ that is $\frac{3}{4}$ of the way from $P$ to $Q$ .
9	Let $\mathbf{v} = (1, -2, 3, 4)$ and $\mathbf{u} = (-7, -2, 4, 5)$ . Check whether $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$
699571	Find the distance $D$ between the point $(-1, 7, -3)$ and the plane $2x + 6y + 3z = 5$ .

## Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks)

11	Change the matrix $\begin{bmatrix} 1 & 8 & 10 \\ 3 & 4 & 5 \\ 9 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ to reduced row echelon form
12	Let $\mathbf{0}$ denote a $2 \times 2$ matrix, each of whose entries is zero. 1. Is there a $2 \times 2$ matrix $A$ such that $A \neq \mathbf{0}$ and $AA = \mathbf{0}$ ? Justify your answer. 2. Is there a $2 \times 2$ matrix $A$ such that $A \neq \mathbf{0}$ and $AA = A$ ? Justify your answer.
13	Using Row Operations to find $A^{-1}$ , $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$
14	Determine whether the homogeneous system has nontrivial solutions $\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 2x_1 + 5x_2 + 3x_3 &= 0 \\ x_1 + 8x_3 &= 0 \end{aligned}$
15	Show that $\det(A) = \begin{vmatrix} \text{tr}(A) & 1 \\ \text{tr}(A^2) & \text{tr}(A) \end{vmatrix}$ , for every $2 \times 2$ matrix
16	Using Cramer's rule solve: $\begin{aligned} x + y + 2z &= 4 \\ 2x - y + 3z &= 9 \\ 3x - y - z &= 2 \end{aligned}$
17	Find vector and parametric equations of the plane in $\mathbb{R}^4$ that passes through the point $x_0 = (2, -1, 0, 3)$ and is parallel to both $v_1 = (1, 5, 2, -4)$ and $v_2 = (0, 7, -8, 6)$ .
18	Show that $\mathbf{u} = (-2, 3, 1, 4)$ and $\mathbf{v} = (1, 2, 0, -1)$ are orthogonal. Verify the Theorem of Pythagoras for these vectors

## Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19 Test for consistency and solve

$$5x + 3y + 3z = 48$$

$$2x + 6y - 3z = 18$$

$$8x - 3y + 2z = 21$$

20 Without evaluating the determinants directly, show that

$$\begin{vmatrix} a_1 & b_1 + ta_1 & c_1 + rb_1 + sa_1 \\ a_2 & b_2 + ta_2 & c_2 + rb_2 + sa_2 \\ a_3 & b_3 + ta_3 & c_3 + rb_3 + sa_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$