D 132038 **(Pages : 4)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define vector space.
2. Define linear transformation between two vectors spaces. Illustrate with an example.
3. Define determinant as a real function on the set of all ordered n-tuples of vectors in $\mathbb{R}^n$.
4. Define parametrized curve in $\mathbb{R}^n$. Give a parametrization of the parabola $x^2 + y^2 = 1$.
5. Compute the curvature of the curve

$$\gamma(t) = (t, \cosh t).$$

6. Describe briefly : Stereographic projection from S^2 to the plane.
7. Let $\sigma(u, v)$ be a surface patch with standard unit normal $N(u, v)$. Then prove that $N_u \cdot \sigma_v = -M$.
8. Describe briefly : Gaussian curvature and mean curvature.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Prove that $\dim \mathbb{R}^n = n$.

10. Suppose f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, f is differentiable in E, and there is a real number M such that

$$\| f'(x) \| \leq M$$

for every $x \in E$. Then prove that

$$| f(b) - f(a) | \leq M | b - a |$$

for all $a \in E$ and $b \in E$.

11. If I is the identity operator on $\mathbb{R}^n$, then prove that

$$\det [I] = \det (e_1, ..., e_n) = 1.$$

MODULE II

12. Find the tangent vector of the limacon

$$\gamma(t) = ((1 + 2\cos t)\cos t, (1 + 2\cos t)\sin t).$$

What is the tangent vector of this curve at the origin ?

13. Let $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ be a unit-speed curve, let $s_0 \in (\alpha, \beta)$ and let φ_0 be such that

$$\dot{\gamma}(s_0) = (\cos \varphi_0, \sin \varphi_0).$$

Then prove that there is a unique smooth function $\varphi : (\alpha, \beta) \to \mathbb{R}$ such that $\varphi(s_0) = \varphi_0$ and that the equation

$$\dot{\gamma}(s) = (\cos \varphi(s), \sin \varphi(s)).$$

holds for all $s \in (\alpha, \beta)$.

14. Let $\sigma : U \to \mathbb{R}^3$ be a patch of a surface $\mathcal{S}$ containing a point $p \in \mathcal{S}$, and let (u, v) be co-ordinates in U. Then prove that the tangent space to $\mathcal{S}$ at p is the vector subspace of $\mathbb{R}^3$ spanned by the vectors σ_u and σ_v.

Module III

15. Prove that the area of a surface patch is unchanged by reparametrization.

16. If $\boldsymbol{\sigma}$ is a surface patch of $\mathcal{S}$ and $\gamma(t) = \sigma(u(t),\ v(t))$ is a curve in $\boldsymbol{\sigma}$, then find κ_n.

17. Prove that the principal curvatures at a point of a surface are the maximum and minimum values of the normal curvature of all curves on the surface that pass through the point. Also prove that the principal vectors are the tangent vectors of the curves giving these maximum and minimum values.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. (a) If $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$, and c is a scalar, then prove that

$$\| A + B \| \leq \| A \| + \| B \|$$

and

$$\| cA \| = | c | \ \| A \|.$$

(b) Suppose f maps on an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. Then prove that $f \in \mathcal{C}'(E)$ if and only if the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m,\ 1 \leq j \leq n$.

19. (a) Prove that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

(b) If $f : \mathcal{S}_1 \to \mathcal{S}_2$ is a diffeomorphism, then prove that for all $p \in \mathcal{S}_1$ the linear map $D_p f : T_p\mathcal{S}_1 \to T_{f(p)}\mathcal{S}_2$ is invertible.

20. Prove that a local diffeomorphism $f : \mathcal{S}_1 \to \mathcal{S}_2$ is conformal if and only if there is a function $\lambda : \mathcal{S}_1 \to \mathbb{R}$ such that

$$f^* \langle v, w \rangle_p = \lambda(p) \langle v, w \rangle_p$$

for all $p \in \mathcal{S}_1$ and $v,\ w \in \mathrm{T}_p\mathcal{S}_1$.

21. (a) Prove that a local diffeomorphism $f : \mathcal{S}_1 \to \mathcal{S}_2$ is a local isometry if and only if, for any surface patch σ_1 of $\mathcal{S}_1,$ the patches $\boldsymbol{\sigma}_1$ and $f \circ \sigma_1$ of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, have the same first fundamental form.

(b) Let $\mathcal{S}$ be a (connected) surface of which every point is an umbilic. Then, prove that $\mathcal{S}$ is an open subset of a plane or a sphere.

(2 × 5 = 10 weightage)

D 132038-A **(Pages : 6)** **Name.......................................**

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Mathematics

MTH 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2019 Admission onwards)

[Improvement Candidates need not appear for MCQ Part]

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(Multiple Choice Questions for SDE Candidates)

1. If matrix of A, $[A]$ has two equal columns, then :

(A) $\det[A] = 1$.
(B) $\det[A] = -1$.
(C) $\det[A] = 0$.
(D) $\det[A^2] = 1$.

2. A linear operator A on $\mathbb{R}^n$ is invertible if and only if :

(A) $\det[A] = 0$.
(B) $\det[A] \neq 0$.
(C) $\det[A] \neq 1$.
(D) $\det[A^{-1}] = -1$.

3. Let $A : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $A(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$ then :

(A) $[A'(1,0)] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

(B) $[A'(1,0)] = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

(C) $[A'(1,0)] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

(D) $[A'(1,0)] = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

4. A parametrization of the parabla $y = x^2$; $x \in \mathbb{R}$ is :

(A) $\gamma(t) = (t, t^2)$; $t \in \mathbb{R}$.
(B) $\gamma(t) = (t, 2t)$; $t \in \mathbb{R}$.
(C) $\gamma(t) = (t^2, t^4)$; $t \in \mathbb{R}$.
(D) $\gamma(t) = (2t, 2t^2)$; $t \in \mathbb{R}$.

5. Speed of the curve $\gamma(t)=\left(\frac{4}{5}\cos t, 1-\sin t, -\frac{3}{5}\cos t\right)$ is :

(A) $\sqrt{2t}$. (B) 0.

(C) $2\sqrt{2}$. (D) 1.

6. If $\gamma(t)=\left(\frac{4}{5}\cos t, 1-\sin t, -\frac{3}{5}\cos t\right)$ then $\dot{\gamma}\cdot\ddot{\gamma}=$

(A) 0. (B) 1.

(C) $\sqrt{2t}$. (D) $2\sqrt{2}$.

7. Speed of the curve $\gamma(t)=\left(t, t^2, t^3\right)$ is :

(A) 1. (B) $\sqrt{1+4t^2+9t^4}$.

(C) 0. (D) $\sqrt{2t}$.

8. Any regular plane curve γ whose curvature is a positive constant is a part of a :

(A) Circle. (B) Straight line.

(C) Cycloid. (D) Sphere.

9. The unit tangent vector **t** of the parametrized curve $\gamma(t)=\left(\frac{4}{5}\cos t, 1-\sin t, -\frac{3}{5}\cos t\right)$ is :

(A) $\mathbf{t}=\left(-\frac{4}{5}\cos t, \sin t, \frac{3}{5}\cos t\right)$. (B) $\mathbf{t}=\left(-\frac{4}{5}\sin t, -\cos t, \frac{3}{5}\sin t\right)$.

(C) $\mathbf{t}=\left(-\frac{3}{5}, 0, -\frac{4}{5}\right)$. (D) $\mathbf{t}=(0, 0, 0)$.

10. The torsion of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta)$ is :

(A) $\frac{b}{a^2+b^2}$.

(B) $\frac{a}{a^2+b^2}$.

(C) $\frac{1}{a^2+b^2}$.

(D) $\frac{b}{a^2+b^2}$.

11. Let γ be a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion.Then, γ is a parametrization of (part of) a :

(A) Plane.

(B) Circle.

(C) Sphere.

(D) Cylinder.

12. A parametrization of the circular cylinder of radius 1 and axis the z - axis is :

(A) $\sigma(u, v) = (v, \cos u, \sin u)$.

(B) $\sigma(u, v) = (\cos u, v, \sin u)$.

(C) $\sigma(u, v) = (\cos u, \sin u, v)$.

(D) $\sigma(u, v) = (\cos u, \sin u, \cos v)$.

13. First fundamental form of the surface patch $\sigma(u, v) = (u - v, u + v, u^2 + v^2)$ is :

(A) $(2 + 4u^2)\,du^2 + 8uv\,dudv + (2 + 4v^2)\,dv^2$.

(B) $du^2 + dv^2$.

(C) $(1 + 4u^2)\,du^2 + 8uv\,\,dudv + (1 + 4v^2)\,dv^2$.

(D) $(v^2 + u^2)(du^2 + dv^2)$.

14. $\| \sigma_u \times \sigma_v \| =$

(A) $\left(FG - E^2\right)^{1/2}$.

(B) $\left(EF - G^2\right)^{1/2}$.

(C) $\left(EG - F^2\right)^{1/2}$.

(D) $\left(EF - G^2\right)$.

15. Let **p** be a point of a surface $\mathcal{S}$, let $\sigma(u, v)$ be a surface patch with standard unit normal $\mathbf{N}(u, v)$ and let $L du^2 + 2M\, du dv + N\, dv^2$ be the second fundamental form of $\boldsymbol{\sigma}$ then :

(A) $\mathbf{N}_u \cdot \sigma_v = LN - M^2$.

(B) $\mathbf{N}_u \cdot \sigma_v = - M$.

(C) $\mathbf{N}_u \cdot \sigma_v = - N$.

(D) $\mathbf{N}_u \cdot \sigma_v = - L$.

16. Every subset of a linearly independent set is :

(A) Linearly independent.

(B) A basis of a Vector space.

(C) Linearly dependent.

(D) Contains null vector.

17. Let κ_g be the geodesic curvature of $\boldsymbol{\gamma}$ then :

(A) $\kappa_g = \ddot{\gamma} \times \mathbf{N}$.

(B) $\kappa_g = \ddot{\gamma} \times (\mathbf{N} \times \dot{\gamma})$.

(C) $\kappa_g = \ddot{\gamma} \cdot (\mathbf{N} \times \dot{\gamma})$.

(D) $\kappa_g = \ddot{\gamma} \cdot \mathbf{N}$.

18. Let $\boldsymbol{\sigma}$ be a surface patch $\mathcal{S}$ with second fundamental form $L du^2 + 2M\, du dv + N\, dv^2$ and let $\gamma(t) = \sigma(u(t), v(t))$ be a curve in $\boldsymbol{\sigma}$, then the normal curvature $\boldsymbol{\kappa}_n$ of $\boldsymbol{\gamma}$ is :

(A) $\kappa_n = L\dot{v}^2 + 2M\dot{v}\dot{u} + N\dot{u}^2$.

(B) $\kappa_n = L\dot{u}\dot{v} + M\dot{u}\dot{v} + M\dot{v}\dot{u} + N\dot{v}\dot{u}$.

(C) $\kappa_n = L\dot{u} + 2M\dot{u}\dot{v} + N\dot{v}$.

(D) $\kappa_n = L\dot{u}^2 + 2M\dot{u}\dot{v} + N\dot{v}^2$.

Turn over

19. Let σ be a surface patch of an oriented surface $\mathcal{S}$ with first and second fundamental forms $Edu^2 + 2F\,dudv + Gdv^2$ and $Ldu^2 + 2M\,dudv + N\,dv^2$, respectively. Then the Gaussian curvature K of $\mathcal{S}$ at **p** is :

(A) $K = \dfrac{LN - M^2}{EG - F^2}$.

(B) $K = \dfrac{LG - 2MF + NE}{EG - F^2}$.

(C) $K = \dfrac{LN - M^2}{2\left(EG - F^2\right)}$.

(D) $K = \dfrac{LG - 2MF + NE}{2\left(EG - F^2\right)}$.

20. The Gaussian curvature of unit cylinder is :

(A) Positive or zero.

(B) 1.

(C) 0.

(D) Negative or zero.