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D 132039 **(Pages : 5)** **Name.......................................**

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. If $f : G \to \mathbb{C}$ is differentiable at a point a in G then prove that f is continuous at a.

2. If S is a Möbius transformation then prove that S is the composition of translations, dilations, and the inversion.

3. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$.

4. Let γ be a rectifiable curve in $\mathbb{C}$ and suppose that F_n and F are continuous functions on $\{\gamma\}$. If

$$F = u - \lim F_n$$

on $\{\gamma\}$ then prove that

$$\int_{\gamma} \lim F_n = \lim \int_{\gamma} F_n.$$

5. If $\gamma : [0, 1] \to \mathbb{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that

$$\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$$

is an integer.

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6. Evaluate $\int_{\gamma} \frac{2z}{z^2 + z + 1} dz$ where γ is the circle $|z| = 2$.

7. Prove that a function $f : [a, b] \to \mathbb{R}$ is convex if and only if the set

$$A = \{(x, y) : a \le x \le b \text{ and } f(x) \le y\}$$

is convex.

8. State Schwarz's Lemma.

(8 × 1 = 8 weightage)

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Let $\sum a_n (z-a)^n$ and $\sum b_n (z-a)^n$ be power series with radius of convergence $\ge r > 0$. Put

$$c_n = \sum_{k=0}^{n} a_k b_{n-k} ;$$

then prove that both power series

$$\sum (a_n + b_n)(z-a)^n$$

and

$$\sum c_n (z-a)^n$$

have radius of convergence $\ge r$, and for $|z - a| < r$ the following hold :

$$\sum (a_n + b_n)(z-a)^n = \left[\sum a_n (z-a)^n + \sum b_n (z-a)^n\right]$$

$$\sum c_n (z-a)^n = \left[\sum a_n (z-a)^n\right]\left[\sum b_n (z-a)^n\right].$$

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10. Let G and Ω be open subsets of $\mathbb{C}$. Suppose that $f : G \to \mathbb{C}$ and $g : \Omega \to \mathbb{C}$ are continuous functions such that $f(G) \subset \Omega$ and $g(f(z)) = z$ for all z in G. If g is differentiable and $g'(z) \neq 0$, then prove that f is differentiable and

$$f'(z) = \frac{1}{g'(f(z))}.$$

11. If a Möbius transformation T takes a circle Γ_1 onto the circle Γ_2 then prove that any pair of points symmetric with respect to Γ_1 are mapped by T onto a pair of points symmetric with respect to Γ_2.

Module II

12. State and prove the Fundamental Theorem of Algebra.

13. If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point $a \in G$ with

$$|f(a)| \geq |f(z)| \text{ for all } z \in G,$$

then prove that f is constant.

14. Let G be simply connected and let $f : G \to \mathbb{C}$ be an analytic function such that $f(z) \neq 0$ for any z in G. Then prove that there is an analytic function $g : G \to \mathbb{C}$ such that $f(z) = \exp g(z)$.

Module III

15. If f has an isolated singularity at a then prove that $z = a$ is a removable singularity if and only if $\lim\limits_{x \to a} (z - a) f(z) = 0$.

16. Let $z = a$ be an isolated singularity of f and let

$$f(z) = \sum_{-\infty}^{\infty} a_n (z - a)^n$$

be its Laurent Expansion in ann $(a ; 0, R)$. Then prove that $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \leq -1$.

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17. Show that

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\pi}{\sqrt{2}}.$$

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. For a given power series

$$\sum_{n=0}^{\infty} a_n (z-a)^n$$

define the number $R, 0 \le R \le \infty$, by

$$\frac{1}{R} = \limsup |a_n|^{1/n}.$$

Then prove that

(a) if $|z-a| < R$, the series converges absolutely;

(b) if $|z-a| > R$, the terms of the series become unbounded and so the series diverges;

(c) if $0 < r < R$ then the series converges uniformly on $\{z : |z| \le r\}$.

19. Let G be an open subset of the plane and $f : G \to C$ an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma ; w) = 0$ for all $w \in C \sim G$, then prove that for $a \in G \sim \{\gamma\}$,

$$n(\gamma ; a) f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{z-a}\, dz.$$

20. State and prove Goursat's Theorem.

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21. (a) State and prove Casorati-Weierstrass theorem.

(b) Let f be analytic in the region G except for the isolated singularities $a_1, a_2, ..., a_m$. If γ is a closed rectifiable curve in G which does not pass through any of the points a_k and if $\gamma \approx 0$ in G. Then prove that

$$\frac{1}{2\pi i}\int_{\gamma} f = \sum_{k=1}^{m} n(\gamma ; a_k) \operatorname{Res}(f, a_k).$$

(2 × 5 = 10 weightage)

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D 132039–A **(Pages : 7)** **Name.......................................**

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH3C12—COMPLEX ANALYSIS

(2019 Admission onwards)

[Improvement Candidates need not appear for MCQ Part]

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

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MTH3C12—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. If G be an open set in $\mathbb{C}$ and $f : G \to \mathbb{C}$ then f is differentiable at a point a in G if ————.

   (A) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h^2}$ exists.

   (B) $\lim_{h \to 0} \frac{f(a+h) + f(a)}{h}$ exists.

   (C) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists.

   (D) None of the above options.

2. A function $f : G \to \mathbb{C}$ is analytic if ————.

   (A) f is continuous on G.

   (B) f is continuously differentiable on G.

   (C) f is differentiable at some points on G.

   (D) None of the above options.

3. If the series $\sum_{n=0}^{\infty} a_n (z-a)^n$ has radius of convergence R > 0 then ————.

   (A) $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ is analytic in B (a ; R).

   (B) $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ is analytic on $\mathbb{C}$.

   (C) $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ need not be analytic on B (a ; R).

   (D) None of the above options.

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4. A function f is periodic with period c if ————.

   (A) $f(z+c) = f(z)$ for all z in $\mathbb{C}$.

   (B) $f(z+c) = f(z)$ for some z in $\mathbb{C}$.

   (C) $f(z+c) = k$ (where k is constant) for all z in $\mathbb{C}$.

   (D) None of the above options.

5. The derivative of a branch of the logarithm function is ————.

   (A) z.

   (B) z^{-2}.

   (C) z^{-1}.

   (D) None of the above options.

6. Which one of the following statements is false :

   (A) $f(z) = e^z$ is conformal throughout $\mathbb{C}$.

   (B) $f(z) = e^z$ is continuous throughout $\mathbb{C}$.

   (C) $f(z) = e^z$ is differentiable throughout $\mathbb{C}$.

   (D) $f(z) = e^z$ is not conformal throughout $\mathbb{C}$.

7. Which one of the following is a translation.

   (A) $S(z) = z + a$ for some complex number a.

   (B) $S(z) = az$ for some complex number a.

   (C) $S(z) = 3z$.

   (D) None of the above options.

8. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by :

$$S(z) = \frac{z - z_3}{z - z_4} \text{ if } z_2 = \infty.$$

   Then $S(z_2) =$ ————.

   (A) 1.

   (B) 0.

   (C) -1.

   (D) None of the above options.

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9. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by :

$$S(z) = \frac{z - z_3}{z - z_4} \text{ if } z_2 = \infty.$$

Then $S(z_4) =$ ————.

(A) 1. (B) 0.

(C) ∞. (D) None of the above options.

10. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $S : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by :

$$S(z) = \frac{z_2 - z_4}{z - z_4} \text{ if } z_3 = \infty.$$

Then $S(z_4) =$ ————.

(A) 1. (B) 0.

(C) –1 . (D) None of the above options.

11. $(z, 1, 0, \infty) =$ ————.

(A) 1. (B) 0.

(C) – 1. (D) None of the above options.

12. If $\sigma : [a, b] \to \mathbb{C}$ is also of bounded variation and $\alpha, \beta \in \mathbb{C}$ then ————.

(A) $V(\alpha\gamma + \beta\sigma) \le \alpha V(\gamma) - \beta V(\sigma)$.

(B) $V(\alpha\gamma + \beta\sigma) \le \alpha V(\gamma) + \beta V(\sigma)$.

(C) $V(\alpha\gamma + \beta\sigma) \le |\alpha| V(\gamma) + |\beta| V(\sigma)$.

(D) $V(\alpha\gamma + \beta\sigma) \le |\alpha| V(\gamma) - |\beta| V(\sigma)$.

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13. Let f and g be continuous functions on $[a, b]$ and let γ and σ be functions of bounded variation on $[a, b]$. Then for any scalars α and β, ————.

(A) $\int_a^b (\alpha f + \beta g)\, d\gamma = |\alpha| \int_a^b f\, d\gamma - |\beta| \int_a^b g\, d\gamma.$

(B) $\int_a^b (\alpha f + \beta g)\, d\gamma = \alpha \int_a^b f\, d\gamma - \beta \int_a^b g\, d\gamma.$

(C) $\int_a^b (\alpha f + \beta g)\, d\gamma = |\alpha| \int_a^b f\, d\gamma + |\beta| \int_a^b g\, d\gamma.$

(D) $\int_a^b (\alpha f + \beta g)\, d\gamma = \alpha \int_a^b f\, d\gamma + \beta \int_a^b g\, d\gamma.$

14. Let γ be a rectifiable curve and suppose that f is a function continuous $\{\gamma\}$. Then ————.

(A) $\int_\gamma f' = -\int_{-\gamma} f.$

(B) $\int_\gamma f = \int_{-\gamma} f.$

(C) $\int_\gamma f = -\int_{-\gamma} f.$

(D) None of the above options.

15. If $|z| < 1$ then $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z}\, ds =$ ————

(A) 2π.

(B) -2π.

(C) $2\pi i$.

(D) $-\pi$.

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16. If f and g are analytic on a region G then ________.

(A) $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ has no limit point in G.

(B) $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ for all z in G.

(C) $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ has a limit point in G.

(D) None of the above options.

17. Let G be an open subset of the plane and $f : G \to \mathbb{C}$ is analytic. If $\gamma_1, \gamma_2, \ldots \gamma_m$ are closed rectifiable curves in G such that :

$n(\gamma_1 ; w) + n(\gamma_2 ; w) + \ldots + n(\gamma_n ; w) = 0$ for all $w \in \mathbb{C} \sim G$, then ________.

(A) $\sum_{k=1}^{m} \int_{\gamma k} f(z)\, dz = 2\pi i.$

(B) $\sum_{k=1}^{m} \int_{\gamma k} f(z)\, dz = 2\pi.$

(C) $\sum_{k=1}^{m} \int_{\gamma k} f(z)\, dz = 0.$

(D) $\sum_{k=1}^{m} \int_{\gamma k} f(z)\, dz = 1.$

18. If $f : G \to \mathbb{C}$ is an analytic function and γ is a closed rectifiable curve in G such that $\gamma \sim 0$, then $\int_{\gamma} f =$ ________.

(A) 1.

(B) π .

(C) 2π .

(D) 0.

19. If γ is the circle $|z| = 2$, then $\int_{\gamma} \frac{2z}{z^2 + z + 1} dz =$ ________.

(A) 0.

(B) 1.

(C) π .

(D) 2π.

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20. Suppose that f has a pole at $z = a$. Then $\lim_{z \to a} |f(z)| =$ ————.

(A) ∞.

(B) 0.

(C) 1.

(D) $-\infty$.

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