**D 132041** **(Pages : 4)** Name.....................................

Reg. No.................................

## THIRD SEMESTER M.Sc. (CBCSS) DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

Mathematics

MTH3C14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

### Part A (Short Answer Type Questions)

*Answer* **all** *questions.*
*Each question carries a weightage* 1.

1. Define a quasilinear first-order PDE.

2. State the formula for the characteristic curves of the quasilinear equation

   $$a(x, y, u)\, u_x + b(x, y, u)\, u_y = c(x, y, u).$$

3. Define the term Eikonal Equation.

4. When an equation

   $$\mathrm{L}[u] - au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$$

   is elliptic. Give an example of an elliptic equation.

5. Let $u(x, t)$ be the solution of the Cauchy problem

   $$u_{tt} - 9u_{xx} = 0, -\infty < x < \infty, t > 0,$$

   with initial conditions

   $$u(x, 0) = f(x) = \begin{cases} 1 & \text{if } |x| \le 2, \\ 0 & \text{if } |x| > 2, \end{cases}$$

   $$u_t(x, 0) = g(x) = \begin{cases} 1 & \text{if } |x| \le 2, \\ 0 & \text{if } |x| > 2. \end{cases}$$

   Find $u\left(0, \frac{1}{6}\right)$.

**Turn over**

6. Define Neumann problem.

7. Define linear integral equation. Give an example.

8. Transform the differential equation with initial conditions :

$$\left.\begin{array}{l}\frac{d^2y}{dx^2} + \lambda y = f(x), \\ y(0) = 1,\ y'(0) = 0\end{array}\right\}$$

into an integral equation.

(8 × 1 = 8 weightage)

## Part B (Paragraph Type Questions)

*Answer any* **two** *questions from each module.*

*Each question carries a weightage 2.*

### Module I

9. Use the method of characteristics to solve the equation $u_x + 2u_y = u^2$.

10. Prove the following :

$$u_x = c_0 u + c_1$$

where $c_0$ is a constant and $c_1$ is a function of $x$ and $y$ has infinitely many solutions when $c_1 = 0$ and

$$u(x, 0) = 2e^{c_0 x}.$$

11. Reduce the equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$ to its canonical form.

### Module II

12. Solve the problem :

$$u_{tt} - 4u_{xx} = 0 < x < 1, t > 0,$$

with boundary conditions

$$u_x(0, t) = u_x(1, t) = 0, t \geq 0,$$

and initial conditions

$$u(x, 0) = f(x) = \cos^2(\pi x), 0 \leq x \leq 1,$$

$$u_t(x, 0) = g(x) = \sin^2(\pi x)\cos(\pi x), 0 \leq x \leq 1.$$

13. Describe the method of separation of variables as applied to the heat equation with homogeneous boundary conditions.

14. State and prove the Maximum Principle for the heat equation.

MODULE III

15. Prove that :

$$\overbrace{\int_a^x \dots \int_a^x}^{n \text{ times}} f(x) \overbrace{dx \dots dx}^{n \text{ times}} = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi) d\xi.$$

16. Transform :

$$\left.\begin{array}{l} \dfrac{d^2y}{dx^2} + \lambda y = 0, \\ y(0) = 0,\ y(l) = 0 \end{array}\right\}$$

to an integral equation.

17. Prove that :

$$y(x) = \lambda \int_0^1 (1 - 3x\xi)\, y(\xi) d\xi$$

has only the trivial solution if and only if $\lambda \neq \pm 2$.

(6 × 2 = 12 weightage)

**Part C (Essay Type Questions)**

*Answer* **two** *questions.*

*Each question carries a weightage* 5.

18. (a) State and prove the formula for the solution of the Cauchy problem for the one-dimensional nonhomogeneous wave equation.

(b) Illustrate the concepts of the domain of dependence and region of influence.

**Turn over**

19. Classify the second-order linear equation :

$$u_{xx} + bu_{xy} + cu_{yy} + du_x + eu_y + fu = g$$

based on the sign of $b^2 - 4ac$ and discuss the reduction to the canonical form for the parabolic case.

20. Derive the solution of the heat equation $u_t = ku_{xx}$ for a rod of length L with homogeneous boundary conditions $u(0, t) = 0$, $u(L, t) = 0$ and initial condition $u(x, 0) = f(x)$, using the method of separation of variables.

21. Prove that the relation :

$$y(x) = \int_a^b G(x, \xi)\, \Phi(\xi)\, d\xi \qquad (1.48)$$

where

$$G(x, \xi) = \begin{cases} -\dfrac{1}{A} u(x)\, v(\xi) & \text{when } x < \xi, \\ -\dfrac{1}{A} u(\xi)\, v(x) & \text{when } x > \xi, \end{cases}$$

implies the differential equation

$$Ly + \Phi(\xi) = 0.$$

(2 × 5 = 10 weightage)