D 132045 **(Pages : 4)** **Name....................................**

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define a random variable.
2. Distinguish between a discrete random variable and a continuous random variable.
3. Define the moment generating function of a random variable X
4. Define n-dimensional RV.
5. Define the covariance of two random variables X and Y.
6. What does it mean for two random variables to be independent.
7. Define convergence in law of sequence of distribution functions.
8. State the Central Limit Theorem.

(8 × 1 = 8 weightage)

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Prove that the RV X defined on the probability space$\left(\Omega, \mathcal{S}, \mathrm{P}\right)$ induces a probability space $\left(\mathcal{R}, \mathfrak{B}, \mathrm{Q}\right)$ by means of the correspondence.

$$\mathrm{Q}(\mathrm{B}) = \mathrm{P}\left\{\mathrm{X}^{-1}(\mathrm{B})\right\} = \mathrm{P}\left\{w : \mathrm{X}(w) \in \mathrm{B}\right\} \quad \text{for all } \mathrm{B} \in \mathfrak{B}.$$

Turn over

10. Let X be an RV and g be a Borel-measurable function on $\mathcal{R}$. Let $Y = g(X)$. If X is of discrete type, then prove that

$$EY = \sum_{j=1}^{\infty} g(x_j) P(X = x_j).$$

in the sense that if either side of the equation exists, so does the other, and then the two are equal. Also prove that, if X is of continuous type with PDF f, then $EY = \int g(x) f(x) \, dx$ in the sense that if either of the two integrals converges absolutely, so does the other, and the two are equal.

11. Let X be an RV with a distribution satisfying $n^{\alpha} P\{|X| > n\} \to 0$ as $n \to \infty$ for some $\alpha > 0$. Then prove that $E|X|^{\beta} < \infty$ for $0 < \beta < \alpha$.

Module II

12. Prove that X and Y are independent RVs if and only if

$$M(t_1, t_2) = M(t_1, 0) M(0, t_2) \text{ for all } t_1, t_2 \in \mathbb{R}.$$

13. Prove the following :

(a) The correlation co-efficient ρ between two RVs X and Y satisfies

$$|\rho| \leq 1.$$

(b) The equality $|\rho| = 1$ holds if and only if there exist constants $a \neq 0$ and b such that $P(aX + b = Y) = 1$.

14. Let $Eh(X)$ exist. Then prove that

$$Eh(X) = E[E(h(X) | Y)].$$

Module III

15. If $\sum_{n=1}^{\infty} \text{var}(X_n) < \infty$, then prove that $\sum_{n=1}^{\infty} (X_n - EX_n)$ converges almost surely.

16. Let (X_n) be a sequence of iid RVs with $0 < \text{var}(X_n) = \sigma^2 < \infty$ and common mean μ.

 Let $S_n = \sum_{j=1}^{n} X_j, n = 1, 2, \ldots$ Then prove that for every $x \in \mathbb{R}$,

$$\lim_{n \to \infty} P\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \le x\right) = \lim_{n \to \infty} P\left(\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \le x\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du.$$

17. Let $X_1, X_2, \ldots, X_n$ be iid RVs such that $n^{-1/2} S_n$ has the same distribution for every $n = 1, 2, \ldots$

 Then, prove that if $EX_i = 0, \text{var}(X_i) = 1,$ the distribution of X_i must be N (0, 1).

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Prove that the function F defined in

$$F(x) = Q(-\infty, x] = P\{w : X(w) \le x\} \quad \text{for all } x \in \mathcal{R}$$

 is a DF.

19. Let X be an RV satisfying

$$\frac{P\{|X| > cx\}}{P\{|X| > x\}} \to 0 \text{ as } x \to \infty \text{ for all } c > 1;$$

 then prove that X possesses moments of all orders.

20. Prove that the joint PDF of $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ is given by

$$g\left(x_{(1)}, x_{(2)}, \ldots, x_{(n)}\right) = \begin{cases} n! \prod_{i=1}^{n} f\left(x_{(i)}\right), & x_{(1)} < x_{(2)} < \ldots < x_{(n)}, \\ 0, & \text{otherwise} \end{cases}$$

21. Let us write $f(x)=o(x)$, if $f(x)/x \to 0$ as $x \to 0$. Then prove that

$$\lim_{n \to \infty}\left[1+\frac{a}{n}+o\left(\frac{1}{n}\right)\right]^n = e^a \qquad \text{for every } a.$$

(2 × 5 = 10 weightage)

D 132045–A **(Pages : 6)** **Name.......................................**

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2025

(CBCSS)

Mathematics

MTH3E04—PROBABILITY THEORY

(2019 Admission onwards)

[Improvement Candidates need not appear for MCQ Part]

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH3E04—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. If X is a random variable and $f(x)$ is its p.d.f., $E\left(\frac{1}{X}\right)$ is used to find :

   (A) Arithmetic mean. (B) Harmonic mean.

   (C) Geometric mean. (D) First central moment.

2. If X and Y two independent variables and their expected values are $\bar{X}$ and $\bar{Y}$ respectively, then ————.

   (A) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=0$.

   (B) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=1$.

   (C) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=C$ (constant).

   (D) All the above.

3. A r.v is a ———— function.

   (A) Continuous. (B) Discrete.

   (C) Real valued. (D) None of the above.

4. The first moment about origin is ————.

   (A) Variance. (B) Mean.

   (C) Standard deviation. (D) None of these.

5. The distribution function $F(x)$ lies between ————.

   (A) -1 to 1. (B) 0 to 1.

   (C) $-\infty$ to ∞. (D) None of the above.

6. A continuous r.v X has p.d.f. $f(x) = kx$, $0 < x < 1$, the value of k = ————.

(A) 3. (B) 2.

(C) 1. (D) 4.

7. If (X, Y) is an RV of the continuous type, then marginal p.d.f. of X is ————.

(A) $f_1(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$.

(B) $f_1(x) = \int_{-\infty}^{0} f(x, y)\, dy$.

(C) $f_1(x) = \int_{0}^{\infty} f(x, y)\, dy$.

(D) None of the above.

8. Let (X, Y) be an RV of the discrete type, then conditional PMF of X, given $Y = y_j$ defined as :

(A) $P\{Y = y_j | X = x_i\} = \dfrac{P\{X = x_i, Y = y_j\}}{P\{Y = y_j\}}$.

(B) $P\{Y = y_j | X = x_i\} = \dfrac{P\{X = x_i, Y = y_j\}}{P\{X = x_i\}}$.

(C) $P\{X = x_i | Y = y_i\} = \dfrac{P\{X = x_i, Y = y_j\}}{P\ \{Y = y_j\}}$.

(D) $P\{X = x_i | Y = y_i\} = \dfrac{P\{X = x_i, Y = y_j\}}{P\ \{X = x_i\}}$.

9. Random variables X and Y are independent if and only if ————.

(A) $F(x, y) = F_1(x) + F_2(y)$ for all $(x, y) \in R_2$.

(B) $F(x, y) = F_1(x) - F_2(y)$ for all $(x, y) \in R_2$.

(C) $F(x, y) = F_1(x) / F_2(y)$ for all $(x, y) \in R_2$.

(D) $F(x, y) = F_1(x) * F_2(y)$ for all $(x, y) \in R_2$.

Turn over

10. Let Y be a continuous RV where :

$$f(y) = \begin{cases} 1/18, & -3 \le y \le 15 \\ 0, & \text{otherwise} \end{cases}$$

then for $-3 \le y \le 15$, distribution F (y) = ————.

(A) $\frac{y-3}{18}$.
(B) $\frac{y}{15}$.
(C) $\frac{y-3}{15}$.
(D) $\frac{y+3}{18}$.

11. For any real nubers a and $b, a \le b,$ the probability density function of a continuous RV X is given by ————.

(A) $P\{a \le X \le b\} = \int_a^b f(x)\,dx$.

(B) $P\{a \ge X \ge b\} = \int_a^b f(x)\,dx$.

(C) $(a)\,P\{a \le X \le b\} = 1 - \int_a^b f(x)\,dx$.

(D) $(a)\,P\{a \le X \le b\} = \int_a^b f(x)\,dx - 1$.

12. The second moment about the mean is ————.

(A) Variance.
(B) Mean.
(C) Standard deviation.
(D) None of these.

13. For any random variable, for which mean μ exist, the first moment about the mean is equal to ————.

(A) Zero.
(B) One.
(C) Mean itself.
(D) None of these.

14. If c_1 and c_2 are two constants, then which one is a false statement ?

    (A) $E[c_1 X + c_2] = c_1 E(X) + c_2$.

    (B) $E[c_1 X + c_2 X] = c_1 E(X) + c_2 E(X)$.

    (C) $E[(c_1 + c_2) X] = (c_1 + c_2) E(X)$.

    (D) $E[c_1 X + c_2] = c_1 + c_2$.

15. Let $X_1, X_2, \ldots X_n$ are independent variables, then $F(x_1, x_2, \ldots x_n)$ is equal to ————.

    (A) $F_1(x_1) \cdot F_2(x_2) \ldots, F_n(x_n)$.

    (B) $F_1(x) \cdot F_2(x_2) \ldots, F_n(x)$.

    (C) $F(x_1) \cdot F(x_2) \ldots, F(x_n)$.

    (D) None of these.

16. If $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$, as $n \to \infty$. Then,

    (A) $X_n \xrightarrow{P} 1$.

    (B) $X_n \xrightarrow{P} X$.

    (C) $X_n \xrightarrow{a.s.} 0$.

    (D) $X_n \xrightarrow{P} 0$.

17. If $f(x)$ is a continuous real valued function and $X_n \xrightarrow{P} X$, then :

    (A) $f(X_n) \xrightarrow{P} X$.

    (B) $f(X_n) \xrightarrow{P} X_n$.

    (C) $f(X_n) \xrightarrow{P} f(X)$.

    (D) $f(X_n) = f(X)$.

18. $X_n \xrightarrow{P} X \Rightarrow g(X_n) \xrightarrow{P} g(c)$, if :

    (A) g is differentiable.

    (B) g is discrete.

    (C) g is continuous.

    (D) Both (A) and (B).

19. Suppose $X \sim P(\lambda)$. What is the distribution of $Y = \frac{(X-\lambda)}{\sqrt{\lambda}}$?

(A) $N(\lambda, \sigma)$.
(B) $N(0, 1)$.
(C) $N(\mu, 1)$.
(D) $N(\lambda, 1)$.

20. The Weak Law of Large Numbers states that if a sample of independent and identically distributed random variables, as the sample size grows larger

(A) The sample mean will tend toward the population mean.
(B) The sample standard deviation will tend toward the population standard deviation.
(C) The sample mean equal to the population mean.
(D) The sample standard deviation equal to the population standard deviation.