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FOURTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2026

MATHEMATICS

MAT4CJ204
Basic Linear Algebra

2024 Admission Onwards

Maximum Time: 2 Hours	**Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Define a subspace. Is the set $\{0\}$ a subspace of any vector space V?
2	Verify whether the set $W = \{\mathbf{v} = (x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$.
3	Find the coordinate vector $[x]_B$ of x relative to the given basis $\mathcal{B} = \{b_1, \ldots, b_n\}$, where $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
4	Find a basis for the subspace of $\mathbb{R}^3$ $H = \{(a, b, c) : a = 2b\}$.
5	Explain the concept of orthogonal projection.
6	Define an orthonormal set. Determine whether the vectors $u = (1, 2, -1)$, $v = (2, -1, 0)$ are orthogonal.
7	Verify the Pythagorean theorem for vectors $u = (1, 1, 0)$, $v = (1, -1, 0)$.
8	Determine whether the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ is orthogonally diagonalizable.
9	True or False: "An nn matrix that is orthogonally diagonalizable must be symmetric.". Justify your answer
10	Explain different types of quadratic forms with proper examples

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Given $\mathbf{v}_1$ and $\mathbf{v}_2$ in a vector space V, let $H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Show that H is a subspace of V.
12	Let W be the set of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$ Show that W is a subspace of $\mathbb{R}^4$ with dimension 2.
13	Find bases for the row space, the column space, and the null space of the matrix $$\begin{bmatrix} 1 & 2 & 4 & -1 \\ 3 & 2 & 0 & 9 \\ -3 & 4 & 1 & 0 \end{bmatrix}$$
14	Let $\mathcal{B}$ be the basis of $\mathbb{P}_3$ consisting of the Hermite polynomials $\mathcal{B} = \{1,\ 2t,\ 4t^2-2,\ 8t^3-12t\}$. Let $p(t) = 7 - 12t - 8t^2 + 12t^3$. Find the coordinate vector of p relative to $\mathcal{B}$.
15	Suppose $\mathbf{y}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{y}$ is orthogonal to every $\mathbf{w}$ in Span $\{\mathbf{u}, \mathbf{v}\}$
16	Let $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Write $\mathbf{y}$ as the sum of a vector in Span$\{\mathbf{u}\}$ and a vector orthogonal to $\mathbf{u}$.
17	Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Verify that 2 is an eigenvalue of A and v is an eigenvector. Then orthogonally diagonalize A.
18	Classify the quadratic form $$9x_1^2 - 8x_1x_2 + 3x_2^2$$

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$. (a) Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? (b) How many vectors are in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? (c) Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?
20	Find a **QR** factorization of the matrix $$\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$$