D 140687 **(Pages : 3)** **Name..............................**

Reg. No..............................

SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH2C06—ALGEBRA—II

(2019 Admission onwards)

Time : Three Hours — Maximum Weightage : 30

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Verify whether $6\mathbb{Z}$ is a maximal ideal of the ring $\mathbb{Z}$ of integers.
2. Find the characteristic of the field $\mathbb{Z}_5$ of the integers mod 5.
3. Verify whether $\sqrt{\pi}$ is algebraic over the field $\mathbb{Q}(\pi)$.
4. Find all automorphisms of the field $\mathbb{Q}\left(\sqrt{2}\right)$.
5. Let σ be the automorphism of the field $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$ such that $\sigma\left(\sqrt{2}\right)=\sqrt{2}$ and $\sigma\left(\sqrt{3}\right)=-\sqrt{3}$. Find the fixed field of σ.
6. Find the splitting field of the polynomial x^3-2 over $\mathbb{Q}$.
7. Let $K=\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)$. Find the subgroup of $G(K/\mathbb{Q})$ that correspond to $E=\mathbb{Q}\left(\sqrt{3}\right)$ in the Galois correspondence.
8. Find the splitting field of the polynomial x^5-1 over $\mathbb{Q}$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Let F be a field of characteristic zero. Show that F contains a subfield isomorphic to the field $\mathbb{Q}$ of rationals.

Turn over

10. Let $p(x)$ be an irreducible polynomial in F[x] where f is a field. Show that the ideal generated by $p(x)$ is a maximal ideal in F[x].

11. Find the algebraic closure of the rationals $\mathbb{Q}$ in the field $\mathbb{C}$ of complex numbers.

Module II

12. Let E, E′ be field of order p^n where p is a prime and n is a positive integer. Show that E and E′ are isomorphic fields.

13. Let $F \leq F \leq \overline{F}$ be fields where $\overline{F}$ is the algebraic closure of F. Let E be a splitting field over F. Show that if σ is an automorphism of $\overline{F}$ leaving F fixed then $\sigma(a) \in E$ for all $a \in E$.

14. Let p be a prime and $E = \mathbb{Z}_p(y)$ where y is an indeterminate. Let $t = y^p$ *and* $F = \mathbb{Z}_p(t) \leq E$. Show that E is an algebraic extension of F.

Module III

15. Describe all elements of the Galois group G(K/$\mathbb{Q}$) where $K = \mathbb{Q}\left(\sqrt{2}, \sqrt{3}\right)$.

16. Show that the 8th cyclotomic polynomial $\phi_8(x)$ is $x^4 + 1$.

17. Show that regular 18-gon is not cosntructible by ruler and compass.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

18. (a) Define maximal ideal in a ring.

    (b) Let R be a commutative ring with unity and M be an ideal of R. Show that M is a maximal ideal of R if and only if R/M is a field.

19. (a) Show that the set of all constructible real numbers forms a subfield of the field of reals.

    (b) Show that doubling the cube is an impossible geometric construction.

20. Let F be a finite field of characteristic p. Let $\sigma_p : F \to F$ be defined by

$$\sigma_p(a) = a^p \text{ for all } a \in F.$$

Show that :

(a) σ_p is an automorphism of F.

(b) The fixed field of σ_p is isomorphic to $\mathbb{Z}_p$.

21. (a) Define normal extension of a field.

    (b) Let K be a finite normal extension of a field F and E be an extension of F such that $E \leq K$. Show that :

    (i) K is a finite normal extension of E.

    (ii) G(K/E) is a subgroup of G(K/F).

(2 × 5 = 10 weightage)

D 140687-A **(Pages : 4)** **Name..............................**

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Mathematics

MTH2C06—ALGEBRA—II

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	Total No. of Questions : 20	Maximum : 5 Weightage

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH2C06—ALGEBRA—II

(Multiple Choice Questions for SDE Candidates)

1. Which of the following is a not a group :
   - (A) (R, +).
   - (B) (Z, –).
   - (C) (R*, .).
   - (D) (Q*, .).

2. {1, i, – i, – 1} is ————. Choose the most suitable answer :
   - (A) Semigroup.
   - (B) Subgroup.
   - (C) Cyclic group.
   - (D) Abelian group.

3. Which of the following is an example of Integral Domain ?
   - (A) Z_4.
   - (B) Z_7.
   - (C) Z_6.
   - (D) Z_{10}.

4. Which of the following is not a field ?
   - (A) Z_4.
   - (B) Z_7.
   - (C) Z_5.
   - (D) Z_2.

5. Find characteristic of $Z_3 \times 3Z$:
   - (A) 4.
   - (B) 2.
   - (C) 3.
   - (D) 0.

6. Which of the following is not a Prime ideal of Z ?
   - (A) 5Z.
   - (B) Z_2.
   - (C) 3Z.
   - (D) 2Z.

7. Which of the following is a prime field ?
   - (A) Z.
   - (B) R.
   - (C) Q.
   - (D) C.

8. Example of a principal ideal of Z is :
   - (A) Z_3.
   - (B) Z_4.
   - (C) Q.
   - (D) 2Z.

9. Number of Maximal ideals of a field is :
   - (A) 0.
   - (B) 2.
   - (C) 3.
   - (D) 1.

10. Which of the following regular n-gon is not constructible ?

    (A) 3. (B) 4.

    (C) 5. (D) 7.

11. Find dimension of Q(i) over Q is :

    (A) 1. (B) 3.

    (C) 5. (D) 2.

12. Highest degree of irreducible polynomial over real numbers is :

    (A) 1. (B) 2.

    (C) 3. (D) 4.

13. Let E is $Q\left(\sqrt{3}, \sqrt{7}\right)$ and F is Q. Then index of E over F is :

    (A) 2. (B) 3.

    (C) 4. (D) 1.

14. Find Galois group of the polynomial $x^2 - 3$ over Q :

    (A) Z_2. (B) Z_3.

    (C) R. (D) Z_5.

15. Find the number of elements less than and relatively prime to 10 :

    (A) 3. (B) 5.

    (C) 4. (D) 8.

16. Find the order of the element (1, 2, 3) in $Z_3 \times Z_3 \times Z_5$:

    (A) 9. (B) 2.

    (C) 3. (D) 15.

17. Find the number of units in the ring Z_5 :

    (A) 2. (B) 4.

    (C) 3. (D) 1.

18. Find number of generators of the group Z_{10} :

    (A) 2. (B) 4.

    (C) 3. (D) 1.

Turn over

19. Number of elements in D_4 is :

    (A) 2. (B) 1.

    (C) 8. (D) 4.

20. Number of elements of order 5 in S_4 is :

    (A) 1. (B) 0.

    (C) 5. (D) 4.