D 140689 **(Pages : 2)** **Name.................................**

Reg. No..............................

SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Let X = {1, 2, 3, 4} and $\tau = \{X, \phi, \{1, 2\}, \{1, 2, 3\}\}$. Verify whether τ is a topology on X.
2. Consider $\mathbb{R}$ with cofinite topology. Find the closure of the set $\mathbb{N}$ of all natural numbers in this space.
3. Let X, Y be topological spaces and X × Y be the product space. Let B be open in Y. Verify whether X × B is open in X × Y.
4. Verify whether $\mathbb{R}$ with usual topology is separable.
5. Verify whether $\mathbb{R}$ with discrete topology is connected.
6. Verify whether $\mathbb{R}$ with usual topology is path connected.
7. Give a topology on the reals $\mathbb{R}$ so that it is not a T_2-space.
8. Let $\mathbb{R}$ be the real line and A = {1, 2, 3} and B = {4, 5}. Find a continuous function $f : \mathbb{R} \to [0, 1]$ such that $f(x) = 0$ for all $x \in$ A and $f(x) = 1$ for all $x \in$ B.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question carries a weightage of 2.

Module I

9. Show that if X is a second countable space then every subspace of X is also second countable.
10. Let $f : X \to Y$ be a continuous function. Show that $f\overline{(A)} \subseteq \overline{f(A)}$ for all $A \subseteq X$.
11. Let X, Y be topological spaces and $f : X \to Y$ be a bijection such that f and f^{-1} are open. Show that f is a homeomorphism.

Turn over

Module II

12. Let $f : X \to Y$ be a continuous surjective map. Show that f is a quotient map.
13. Show that continuous image of a connected space is connected.
14. Show that if A is a connected subset of topological space X then $\overline{A}$ is also connected.

Module III

15. Show that in a Hausdorff space limits are unique.
16. Show that every metric space is a normal space.
17. Give an example of a T_1-space which is not a T_2-space.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. (a) Define subbase of a topology.
    (b) Show that $\mathcal{S} = \{(-\infty, a), (b, \infty) : a, b \in \mathbb{R}\}$ is a subbase for the usual topology of $\mathbb{R}$.
    (c) Let $\mathcal{S}$ be a family of subsets of a set X and τ be the smallest topology on X containing $\mathcal{S}$. Show that $\mathcal{S}$ is a subbase for τ.
19. (a) Define interior of a set in a topological space.
    (b) Let X be a topological space and $A \subseteq X$. Show that $\text{int}(A) = \cup\{G \subseteq A : G \text{ is open}\}$.
    (c) Find $\text{int}(\mathbb{Q})$ in the usual topology of $\mathbb{R}$ where $\mathbb{Q}$ is the set of all rationals.
20. (a) Define compact space.
    (b) Show that every continuous image of a compact space is compact.
    (c) Show that every real valued function on compact space is bounded.
21. (a) Define Hausdorff space.
    (b) Show that every Hausdorff space is a T_1-space.
    (c) Show that every compact Hausdorff space is a T_4-space.

(2 × 5 = 10 weightage)

D 140689-A **(Pages : 4)** **Name...................................**

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SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B) and (C) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 2C 08—TOPOLOGY

(Multiple Choice Questions for SDE Candidates)

1. Which of the following statements are true for a metric topology (X, d) ?
   (A) Arbitrary intersection of open set is open.
   (B) Arbitrary union of closed set is open.
   (C) Arbitrary union of open set is open.
2. Which of the following is true for discrete topology on X ?
   (A) The topology coincides with the power set P(X).
   (B) Weaker than indiscrete topology on X.
   (C) Neither of (A) and (B).
3. If U is open in X and A is closed in X, then :
   (A) U/A is open in X.
   (B) U/A is closed in X.
   (C) U/A is both and closed in X.
4. A topological space is said to be second countable if :
   (A) It has a countable base.
   (B) It has countable elements.
   (C) It has a finite base.
5. Let X be a set and S is a family of subset of X and I be a topology on X generated by S. Then :
   (A) S is the base for I.
   (B) S is the subbase for I.
   (C) Neither (A) nor (B) is true.
6. The diagonal $\Delta = \{x \times x : x \in X\}$ is closed in X × X if and only if :
   (A) X is a T_1 space.
   (B) X is a compact space.
   (C) X is a Hausdorff space.

7. Let Y be a subset of X and I be a topology on X. Then the subspace topology ∪ on Y induced by I is defined as :

   (A) $\cup = \{V \in Y : V \in I\}$.

   (B) $\cup = \{V \in Y :$ there exists $U \in I$ s.t $V = U \cap Y\}$.

   (C) None of the above.

8. Let be a topological space (X, I). Then the closure of φ is :

   (A) X.

   (B) Not defined.

   (C) φ.

9. Suppose A is a closed subset. Then $\overline{A}$ is :

   (A) X.

   (B) φ.

   (C) A.

10. What are the dense subsets of a space X with indiscrete topology ?

    (A) φ.

    (B) Any non-empty subset of X.

    (C) only X.

11. Let X be a topological space and $A \subset X$. Then interior of $A(\text{int}(A))$ is :

    (A) Union of all open sets contained in A.

    (B) Largest open subset of X contained in A.

    (C) Both (A) and (B).

12. Let $f : X \to Y$ be a function and U, V be topologies on X, Y respectively. Then f is said to be continuous if :

    (A) For all $V \in V$, $f^{-1}(V) \in U$.

    (B) For all $V \in U$, $f(V) \in U$.

    (C) None of the above.

13. A subset A of a space X is said to be a Lindeloff subset of X if :

    (A) Every cover of A by open subsets of X has a countable subcover.

    (B) Every cover of A by open subsets of X has a finite subcover.

    (C) There exists cover of A by open subsets of X which has a countable subcover.

Turn over

14. Which of the statements below is false ?

    (A) Every path-connected space is connected.

    (B) Subsets of the real line R are connected if and only if they are path connected.

    (C) Topologist's sine curve is path connected.

15. Which of the following is true for Hausdorff space ?

    (A) Limits of sequences are unique.

    (B) Singletons are closed.

    (C) Both (A) and (B).

16. Let $f : X \to Y$ is one-to-one and continuous and Y is Hausdorff, then X is necessarily Hausdorff ?

    (A) True.

    (B) False.

    (C) Need not be.

17. An indiscrete space is :

    (A) Regular but not normal.

    (B) Normal but not regular.

    (C) Both regular and normal.

18. Which of the following statement is true ?

    (A) There exists spaces which satisfies T_4-axion but fails to satisfy T_3-axiom.

    (B) Metric spaces need not be T_3.

    (C) Every Tychonoff space is T_3.

19. Consider the set $K = \{1/n : n = 1, 2, \ldots\} \subset R$. Determine the closure of K when R is endowed with the cofinite topology.

    (A) K.

    (B) $K \cup \{0\}$.

    (C) R.

20. Let S is subset of a topological space R^2 with usual topology. If finitely many points (more than one) are removed from the set $S = \{(x, y) : x^2 + y^2 = 1\}$, then the resulting set is connected :

    (A) True.

    (B) False.

    (C) Cant say.