D 140690 **(Pages : 3)** **Name.....**

Reg. No.....

SECOND SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, APRIL 2026

Mathematics

MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Find the series solution of the differential equation $y' = 2xy$.
2. Is $x = 0$ an ordinary point for the differential equation $\left(1+x^2\right)y'' + 2xy' - 2y = 0$.
3. Find a regular singular point of the following differential equation on the x-axis :

   $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0.$
4. Prove that $\Gamma(n+1) = n!$ for any integer $n \geq 0$.
5. Describe the autonomous system with the help of an example.
6. Show that $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ is a solution of the homogeneous system $\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases}$.
7. Find the exact solution of the initial value problem $y' = y^2, y(0) = 1$.
8. Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on the rectangle $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

(8 × 1 = 8 weightage)

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage of 2.

Module I

9. Solve the differential equation $x^2y'' + (2-x)y' = 0$.
10. Prove that $\lim_{b\to\infty} \mathrm{F}\left(a, b, a, \frac{x}{b}\right) = e^x$.

11. Prove that $P_{2n}(0) = (-1)^n \frac{1.3\ldots(2n-1)}{2^n n!}$.

Module II

12. Prove that $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$.

13. Find the general solution of $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$.

14. Find a Liapunov function for system $\begin{cases} \frac{dx}{dt} = -2x + xy^3 \\ \frac{dy}{dt} = -x^2y^2 - y^3. \end{cases}$

Module III

15. Using Picard's method of successive approximation find first three successive approximate solutions of the initial value problem $y' = x + y$, $y(0) = 0$.

16. Find the shortest curve joining (1, 1) and (3, 4).

17. Let $y(x)$ be a nontrivial solution of equation $y'' + q(x)y = 0$ on a closed interval $[a, b]$. Prove that $y(x)$ has atmost finite number of zeros in this interval.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Prove that the differential equation $4x^2y'' - 8x^2y' + \left(4x^2 + 1\right)y = 0$ has only one Frobenius series solution. Also find the general solution.

19. Find the general solution of the differential equation $\left(x^2 - 1\right)y'' + (5x + 4)y' + 4y = 0$ near the singular point $x = -1$.

20. Prove that $\int_0^1 x J_p(\lambda_m x) J_p(\lambda_n x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} J_{p+1(\lambda_n)^2} & \text{if } m = n, \end{cases}$ where λ_m and λ_n are zeros of the Bessel function $J_p(x)$.

21. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| \leq K|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y)$ $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$.

(2 × 5 = 10 weightage)