D 143203 **(Pages : 3)** **Name..................................**

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH4E05—A DVANCED COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 *weightage.*

1. If a set $\mathscr{F} \subset H(G)$ is locally bounded then prove it is uniformly bounded on every compact subsets of G.

2. Let $\operatorname{Re} z_n > -1$. Prove that the series $\sum \log(1+z_n)$ converges absolutely if and only if $\sum z_n$ converges absolutely.

3. Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$.

4. Let $S = \{z : a \le \operatorname{Re}(z) \le A\}$ where $0 < a < A < \infty$. Show that for every $\in > 0$ there is a $\delta > 0$ such that for all z in S.

$$\left|\int_{\alpha}^{\beta} e^{-t}\, t^{z-t}\, dt\right| < \in$$

whenever $0 < \alpha < \beta < \delta$.

5. Define the polynomial convex hull of K and give an example.

6. Let G be an open connected subset of $\mathbb{C}$. Show that if G is homeomorphic to the unit disk then G is simply connected.

Turn over

7. Define the term function element, germ and analytic continuation of a function element along a path.

8. State Hadamard Factorization theorem.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each modules.*
Each question carries a weightage 2.

MODULE I

9. Suppose $\mathscr{F} \subset C(G, \Omega)$ is equicontinuous at each point of G . Prove that $\mathscr{F}$ is equicon-tinuous over each compact subsets of G.

10. Show that if $\{f_n\}$ is a sequence in H (G) and f belongs to C (G, $\mathbb{C}$) such that $f_n \to f$ then f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$.

11. State and prove Hurwitz's theorem.

(2 × 2 = 4 weightage)

MODULE II

12. Show that the non-negative function f defined on $(0, \infty)$ has the following properties :

(a) $\log f(x)$ is a convex function ;

(b) $f(x + 1) = xf(x)$ for all x ;

(c) $f(1) = 1$.

Then $f(x) = \exp(x)$ for all x.

13. State and prove Euler's theorem.

14. Let G be an open connected subset of $\mathbb{C}$. If $n\ (\gamma\,; a) = 0$ for every closed rectifiable curve γ in G and every point in G, then prove that $\mathbb{C}_\infty - G$ is connected.

(2 × 2 = 4 weightage)

MODULE III

15. Let $\gamma : [0, 1] \mapsto \mathbb{C}$ be a path from a to b and let $\{f_t, D_t) : 0 \le t \le 1\}$ and $\{(g_t, B_t) : 0 \le t \le 1\}$ be analytic continuations along γ such that $[f_0]_a = [g_0]_a$. Show that $[f_1]_b = [g_1]_b$.

16. State and prove Jensen's formula.

17. Let f be an entire function of finite order. Prove that f assumes each complex number with one possible exception.

(2 × 2 = 4 weightage)

Part C

Answer any **two** *questions.*

Each question carries a weightage 5.

18. Show that a family $\mathscr{F} \subset M(G)$ is normal in $C(G, \mathbb{C}_\infty)$ iff $\mu(\mathscr{F}) \equiv \{\mu(\mathscr{F}) : f \in \mathscr{F}\}$ is locally bounded.

19. Let G be a region and let $\{a_j\}$ be a sequence of distinct points in G with no limit point in G ; and let $\{m_j\}$ be a sequence of integers. Show that there is an analytic function f defined on G whose only zeros are at the points a_j; furthermore, a_j is a zero of f of multiplicity m_j.

20. Let K be a compact subset of the region G ; Then show that there are straight line segments $\gamma_1, \gamma_2, \ldots \gamma_n$ in G – K such that for every function f in H (G).

$$f(z) = \sum_{k=1}^{n} \frac{1}{2\pi i} \int_{\gamma k} \frac{f(w)}{w - z} \, dw$$

for all z in K. The line segments form a finite number of polygons.

21. State and prove Schwarz reflection principle .

(2 × 5 = 10 weightage)

D 143203–A **(Pages : 7)** **Name..**

Reg. No.....................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH4E05—ADVANCED COMPLEX ANALYSIS

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH4E05—ADVANCED COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. If G is connected and $\Omega = \mathbb{N} = \{1, 2, ...\}$ and $C(G, \Omega)$ contains__________

   (A) Only one element.
   (B) Only constant functions.
   (C) Only finite number of functions.
   (D) None of the above.

2. Which one of the following statements is true.

   (A) Each element in $C(G, \mathbb{C})$ is an analytic function.
   (B) Each analytic function on G is a constant.
   (C) Each analytic function on G is in $C(G, \mathbb{C})$.
   (D) None of the above.

3. If (S, d) is a metric space and

$$\mu(s,t) = \frac{d(s,t)}{1 + d(s,t)}$$

   then ————.

   (A) A set is open in (S, d) need not imply that it is open in (S, μ).
   (B) A set is open in (S, μ) need not imply that it is open in (S, d).
   (C) A set is open in (S, d) implies it is closed in (S, μ).
   (D) None of the above.

4. If (S, d) is a metric space and :

$$\mu(s,t) = \frac{d(s,t)}{1 + d(s,t)}$$

   then ————.

(A) A sequence is a Cauchy sequence in (S, d) need not imply that it is a Cauchy sequence in (S, μ).

(B) A sequence is a Cauchy sequence in (S, μ) need not imply that it is a Cauchy sequence in (S, d).

(C) A sequence is a Cauchy sequence in (S, d) if and only if it is a Cauchy sequence in (S, μ).

(D) None of the above.

5. A sequence $\{f_n\}$ in $C(G, \Omega)$, ρ) converges to f if and only if ————.

(A) $\{f_n\}$ converges to f uniformly on all compact subsets of G.

(B) $\{f_n\}$ converges to f uniformly for some compact subset of G.

(C) $\{f_n\}$ converges to f on some compact subsets of G.

(D) None of the above.

6. A set $\mathcal{F} \subset C(G, \Omega)$ is normal ————.

(A) If and only if its closure is compact.

(B) If and only if it is compact.

(C) If it contains only one element.

(D) None of the above.

7. A metric space (X, d) is sequentially compact if ————.

(A) Every sequence in X converges to 0.

(B) Every sequence in X is convergent in X.

(C) Every sequence in X has a convergent subsequence in X.

(D) None of the above.

8. If (X_n, d_n) be a metric space for each $n \geq 1$ and let $X = \prod_{n=1}^{\infty} X_n$ be their Cartesian product. For

$\xi = \{x_n\}$ and $\eta = \{y_n\}$ in X, define

$$d(\xi, \eta) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \frac{d_n(x_n, y_n)}{1 + d_n(x_n, y_n)}.$$

Then ————.

(A) d is a metric on X.

(B) d is not a metric on X.

(C) d does not satisfy triangle inequality.

(D) None of the above.

9. Let (X_n, d_n) be a metric space for each $n \geq 1$ and let $X = \prod_{n=1}^{\infty} X_n$ be their Cartesian product. For

$\xi = (x_n)$ and $\eta = \{y_n\}$ in X, define

$$d(\xi, \eta) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \frac{d_n(x_n, y_n)}{1 + d_n(x_n, y_n)}.$$

If each (X_n, d_n) is compact then ————.

(A) X is compact. (B) X need not be compact.

(C) X need not be totally bounded. (D) None of the above.

10. A set $\mathcal{F} \subset C(G, \Omega)$ is equicontinuous over a set $E \subset G$————.

(A) If for every $\varepsilon > 0$ there exists $\delta > 0$ such that

z and $z' \in E$ and $|z - z'| < \delta \Rightarrow d(f(z), f(z')) < \varepsilon$ for every $f \in \mathcal{F}$.

(B) If for every $\varepsilon > 0$ there exists $\delta > 0$ such that

z and $z' \in$ E and $|z - z'| < \delta \Rightarrow d\left(f(z), f(z')\right) = \varepsilon$ for every $f \in \mathcal{F}$.

(C) If for every $\varepsilon > 0$ there exists $\delta > 0$ such that

z and $z' \in$ E and $|z - z'| < \in \Rightarrow d\left(f(z), f(z')\right) < \delta$ for every $f \in \mathcal{F}$.

(D) None of the above

11. $\{f\}$, consisting of single function f, is equicontinuous over a set implies ————.

(A) f is constant on that set.

(B) f is analytic on that set.

(C) f is uniformly continuous on that set.

(D) None of the above.

12. $H(G)$ is ————.

(A) A finite subset of C (G, $\mathbb{C}$).

(B) An open subset of C(G, $\mathbb{C}$).

(C) A closed subset of C (G, $\mathbb{C}$).

(D) None of the above.

13. H (G) is ————.

(A) Is a set of constant functions

(B) A finite set.

(C) A complete metric space.

(D) None of the above.

14. If $\{f_n\} \subset H(G)$ converges to $f \in H(G)$ and each f_n never vanishes on G then ————.

(A) $f \equiv 0$.

(B) f never vanishes.

(C) either $f \equiv 0$ or f never vanishes.

(D) None of the above.

15. A set $\mathcal{F} \subset \mathrm{H}(\mathrm{G})$ is locally bounded if ————.

    (A) For each point $a \in \mathrm{G}$ there is constants $\mathrm{M} > 0$ and $r > 0$ such that $|f(z)| \le \mathrm{M}$, for $|z - a| = r$ and for all $f \in \mathcal{F}$.

    (B) For each point $a \in \mathrm{G}$ there are constants $\mathrm{M} > 0$ and $r > 0$ such that $|f(z)| \le \mathrm{M}$, for $|z - a| < r$ and for all $f \in \mathcal{F}$.

    (C) For each point $a \in \mathrm{G}$ there are constants $\mathrm{M} > 0$ and $r > 0$ such that $|f(z)| = \mathrm{M}$, for $|z - a| < r$ and for all $f \in \mathcal{F}$.

    (D) None of the above.

16. Let G be an open subset of the plane and $f : \mathrm{G} \to \mathrm{C}$ an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma; w) = 0$ for all $w \in \mathrm{C} \sim \mathrm{G}$, then for a $a \in \mathrm{G} \sim (\gamma)$ ————.

    (A) $n(\gamma; a) f(a) = \dfrac{1}{2\pi i} \displaystyle\int_\gamma \frac{f(z)}{z-a}\, dz.$

    (B) $n(\gamma; a) f(a) = \dfrac{1}{\pi i} \displaystyle\int_\gamma \frac{f(z)}{z-a}\, dz.$

    (C) $n(\gamma; a) f(a) = \dfrac{1}{2\pi} \displaystyle\int_\gamma \frac{f(z)}{z-a}\, dz.$

    (D) None of the above.

17. A set $\mathcal{F} \subset \mathrm{H}(\mathrm{G})$ is compact if and only if ————.

    (A) It is locally bounded.

    (B) It is closed.

    (C) It is closed and locally bounded.

    (D) None of the above.

18. If $\rho > 0$ is given then ————.

    (A) There is a compact set $\mathrm{K} \subset \mathbb{C}$ such that $\mathbb{C}_\infty - \mathrm{K} \subset \mathrm{B}_\infty(\infty; \rho)$.

    (B) There is a compact set $\mathrm{K} \subset \mathbb{C}$ such that $\mathbb{C}_\infty - \mathrm{K} = \mathrm{B}_\infty(\infty; \rho)$.

    (C) There is a compact set $\mathrm{K} \subset \mathbb{C}$ such that $\mathbb{C}_\infty - \mathrm{K} \supset \mathrm{B}_\infty(\infty; \rho)$.

    (D) None of the above.

19. If a compact set $K \subset \mathbb{C}$ is given, then ————.

    (A) For any $p > 0, B_{\infty}(\infty ; \rho) \subset \mathbb{C}_{\infty} - K$.

    (B) There is a number $p > 0$ such that $B_{\infty}(\infty ; \rho) \subset \mathbb{C}_{\infty} - K$.

    (C) There is a number $p > 0$ such that $B_{\infty}(\infty ; \rho) = \mathbb{C}_{\infty} - K$.

    (D) None of the above.

20. Let $\{f_n\}$ be a sequence in M (G) and suppose $f_n \to f$ in $C(G, \mathbb{C}_{\infty})$. If each f_n is analytic then ————.

    (A) Either f is analytic or $f \equiv \infty$.

    (B) f is analytic.

    (C) $f \equiv \infty$.

    (D) None of the above.