D 143207 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH 4E 09—DIFFERENTIAL GEOMETRY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Sketch the level set $f^{-1}(c)$ at $c=0$ for the function $f(x_1, x_2) = x_1^2 - x_2^2$.

2. Define parametrized curve in $\mathbb{R}^{n+1}$.

3. Find the spherical image of the cylinder $x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$.

4. For the parametrized curve α given by $\alpha(t) = (\cos t, \sin t)$ find the speed $\| \dot{\alpha} \|$.

5. Show that for $a, b, c, d \in \mathbb{R}$ the parametrized curve

$$\alpha(t) = (\cos(at+b), \sin(at+b), ct+d)$$

is a geodesic on the cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$.

6. Let $\bar{X}$ be a Euclidean parallel vector field along a parametrized curve α. Show that $\dot{\bar{X}} = 0$.

7. Define principal curvature of an oriented n-surface.

Turn over

8. Let $\psi : \mathrm{U} \to \mathbb{R}^3$ be defined by

$$\psi(\theta, \phi) = (r \cos\theta \sin\phi, r \sin\theta \sin\phi, r \cos\phi)$$

where $\mathrm{U} = \{(\theta, \phi) : 0 < \phi < \pi\}$. Find $\mathrm{Im}\psi$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit*
Each question has weightage 2.

Unit I

9. Sketch the vector field $\bar{\mathrm{X}}(p) = (p, \mathrm{X}(p))$ where $\mathrm{X}(p) = p$.

10. Show that $\nabla f(\alpha(t_0)) \cdot \dot{\alpha}(t_0) = 0$ where $\boldsymbol{\alpha}$ is a parametrized curve whose image is contained in $f^{-1}(c)$ and $\alpha(t_0) \in f^{-1}(c)$.

11. Show that the n-plane $a_1x_1 + a_2x_2 + ... + a_{n+1}x_{n+1} = 1$ where $(a_1, a_2, ..., a_{n+1}) \neq 0$ is an n-surface in $\mathbb{R}^{n+1}$.

Unit II

12. Let $\bar{\mathrm{X}}, \bar{\mathrm{Y}}$ be smooth vector fields along a parametrized curve . Show that $(\bar{\mathrm{X}} \cdot \bar{\mathrm{Y}}) = \bar{\mathrm{X}}\,\bar{\mathrm{Y}} + \bar{\mathrm{X}} \cdot \bar{\mathrm{Y}}$.

13. Show that $(\bar{\mathrm{X}} + \bar{\mathrm{Y}})' = \bar{\mathrm{X}}' + \bar{\mathrm{Y}}'$. where $'$ denotes covariant derivative.

14. Let C be a connected oriented plane curve and $\beta : \mathrm{I} \to \mathrm{C}$ be a unit speed parametrization of C. Show that if $\boldsymbol{\beta}$ periodic then C is compact.

Unit III

15. Show that a parametrized 1-surface is a regular parametrized curve.

16. Let $\psi : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$\psi(\theta, \phi) = ((a + b\cos\phi)\cos\theta, (a + b\cos\phi)\sin\theta, b\sin\phi).$$

For $p = (\theta, \phi)$ find $E_1(p)$ and $L_p(E_1(p))$.

17. Define local parametrization and chart associated with it for an n-surface S.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Define integral curve of a vector field.

    (b) Show that every smooth vector field has an integral curve.

19. (a) Define maximal integral curve of a tangent vector field on an n-surface S.

    (b) Let S be an n-surface in $\mathbb{R}^{n+1}$ and $\bar{X}$ be a smooth tangent vector field on S. Show that for each $p \in S$ there exists a maximal integral curve on $\bar{X}$ through p.

20. (a) Define Weingarten map L_p of an n-surface S.

    (b) Show that $L_p(v) \cdot w = v \cdot L_p(w)$ for all $v, w \in S_p$.

21. (a) Define Gauss-Kronecker curvature $K(p)$ of an n-surface S.

    (b) Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$. Show that $K(p) \neq 0$ for all $p \in S$ if and only if the second fundamental form at p is definite for all $p \in S$.

(2 × 5 = 10 weightage)

D 143207-A **(Pages : 6)** **Name.......................................**

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FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH 4E 09—DIFFERENTIAL GEOMETRY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 4E 09—DIFFERENTIAL GEOMETRY

(Multiple Choice Questions for SDE Candidates)

1. The graph of a smooth real valued function in R^n is ——————.

   (A) An n-surface in R^{n+1}.
   (B) A 1-surface in R^n.
   (C) $(n-1)$-surface in R^{n+1}.
   (D) Not a surface.

2. Integral curve of the vector field $X(x_1, x_2) = (1, 0)$ through $(0, 0)$ is ——————.

   (A) The line $x_1 = 0$.
   (B) The line $x_1 = 1$.
   (C) The line $x_2 = 0$.
   (D) The line $x_2 = 1$.

3. $f(x_1, x_2) = x_1^2 - x_2^2$. The level set of f at height 1 is ——————.

   (A) Empty.
   (B) A rectangular hyperbola.
   (C) A point.
   (D) A pair of straight lines.

4. The dimension of the tangent space to an n-surface in R^{n+1} at a point is :

   (A) 1.
   (B) n.
   (C) $n - 1$.
   (D) $n + 1$.

5. $f : U \to R, U \subset R^{n+1}$ be smooth and $\alpha : I \to U$ be a parametrized curve. Then which of the following is a necessary and sufficient condition for the image of α is contained in a level set of ?

   (A) $\alpha(t)$ is a constant.
   (B) $f(\alpha(t)) = 0$.
   (C) $\dot{\alpha}(t) = k\Delta f(\alpha(t))$.
   (D) $\dot{\alpha}(t) \perp \Delta f(\alpha(t))$.

6. What is the surface of revolution obtained by rotation the plane curve $x_2 = 1$ about x_1 – axis :

   (A) Plane.
   (B) Sphere.
   (C) Cylinder.
   (D) Torus.

7. S be an n-surface in $\mathbb{R}^{n+1}$. Then the basis $\{\mathbf{v}_1, \ldots\ldots\ldots, \mathbf{v}_n\}$ for the tangent space S_p is said to be consistent if ————————.

(A) $\det\begin{pmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{pmatrix} \neq 0.$

(B) $\det\begin{pmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{pmatrix} = 0.$

(C) $\det\begin{pmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{pmatrix} > 0.$

(D) $\det\begin{pmatrix} v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{pmatrix} < 0.$

8. The unit n-sphere is disconnected for :

(A) $n > 1.$

(B) $n < 3.$

(C) $n = 1.$

(D) $n = 0.$

9. What is spherical image of a surface ?

(A) Image under Weingarten map.

(B) Image under Gauss map.

(C) Image under Lagrange's map.

(D) Image of a sphere.

10. $\alpha : I \to S$ is a geodesic on an n-surface S in $\mathbb{R}^{n+1}$ if ————————.

(A) α is orthogonal to S.

(B) Speed of α is constant.

(C) $\dot{\alpha} \perp \ddot{\alpha}.$

(D) All are true.

11. Covariant derivative of a vector field X on an n-surface S is given by ——————.

    (A) $\dot{\mathbf{X}}(t)+\{\dot{\mathbf{X}}(t)\cdot\mathbf{N}(\alpha(t))\}\mathbf{N}(\alpha(t))$.

    (B) $\dot{\mathbf{X}}(t)+\{\dot{\alpha}(t)\cdot\mathbf{N}(\alpha(t))\}\mathbf{N}(\alpha(t))$.

    (C) $\dot{\mathbf{X}}(t)-\{\dot{\mathbf{X}}(t)\cdot\mathbf{N}(\alpha(t))\}\mathbf{N}(\alpha(t))$.

    (D) $\dot{\mathbf{X}}(t)-\{\dot{\alpha}(t)\cdot\mathbf{N}(\alpha(t))\}\mathbf{N}(\alpha(t))$.

12. **X** and **Y** be two vector fields along a parametrized curve $\boldsymbol{\alpha}$ on an n-surface S. Then $(\mathbf{X}\cdot\mathbf{Y})' =$ ——————.

    (A) $\dot{\mathbf{X}}\cdot\mathbf{Y}+\mathbf{X}\cdot\dot{\mathbf{Y}}$.
    (B) $\mathbf{X}'\cdot\mathbf{Y}+\mathbf{X}\cdot\mathbf{Y}'$.

    (C) $\mathbf{X}'\cdot\dot{\mathbf{Y}}+\dot{\mathbf{X}}\cdot\mathbf{Y}'$.
    (D) Both (A) and (B).

13. Which of the following is not true ?

    (i) **X** is parallel if and only if **X** is constant.

    (ii) If **X** and **Y** are parallel then the angle between **X** and **Y** is constant.

    (iii) If **X** and **Y** are parallel then **X . Y** is constnat.

    (A) Only (i) is false.
    (B) Only (ii) is false.

    (C) Only (iii) is false.
    (D) Both (ii) and (iii) are false.

14. What is the parallel transport of $\mathbf{v}=(0,0,1,1,0,0)$ from $(0,0,1)$ to $(0,0,-1)$ along the parametrized curve $\alpha_\theta(t)=(\cos\theta\sin t,\sin\theta\sin t,\cos t)$ on $\mathbf{S}^2$?

    (A) $(0,0,-1,\cos\theta,\sin\theta,0)$.
    (B) $(0,0,-1,-\cos\theta,-\sin\theta,0)$.

    (C) $(0,0,-1,2\cos\theta,2\sin\theta,0)$.
    (D) $(0,0,-1,-\cos 2\theta,-\sin 2\theta,0)$.

15. What is the parallel transport of $\mathbf{v} = (0, 0, 1, 1, 0, 0)$ from $(0, 0, 1)$ to $(0, 0, -1)$ along the parametrized curve $\alpha(t) = (0, \sin t, \cos t)$ on $\mathbf{S}^2$?

    (A) $(0, 0, -1, 0, 1, 0)$.
    (B) $(0, 0, -1, 0, -1, 0)$.
    (C) $(0, 0, -1, 0, 2, 0)$.
    (D) $(0, 0, -1, 1, 0, 0)$.

16. Which of the following is true ?

    (i) $(\mathbf{X} \cdot \mathbf{Y})' = \mathbf{X}' \cdot \mathbf{Y} + \mathbf{X} \cdot \mathbf{Y}'$.

    (ii) $(\mathbf{X} \cdot \mathbf{Y})' = \dot{\mathbf{X}} \cdot \mathbf{Y} + \mathbf{X} \cdot \dot{\mathbf{Y}}$.

    (A) Only (i) is true.
    (B) Only (ii) is true.
    (C) Both are false.
    (D) Both are true.

17. The Weingarten map is given by for $\mathbf{v} = \dot{\alpha}(t_0)$, $L_p(\mathbf{v}) =$ ————————————.

    (A) $(\mathbf{N} \circ \alpha)'(t_0)$.
    (B) $-(\mathbf{N} \circ \alpha)'(t_0)$.
    (C) $(\mathbf{N} \dot{\circ} \alpha)(t_0)$.
    (D) $-(\mathbf{N} \dot{\circ} \alpha)(t_0)$.

18. Let S be a plane curve, $p \in S$ such that the radius of curvature at p is non zero, and let C be the circle of curvature at p. Then which of the following statement is true ?

    (i) $S_p = C_p$

    (ii) orientation normals of S and C at p are equal

    (A) Only (i) is true.
    (B) Only (ii) is true.
    (C) Both are false.
    (D) Both are true.

19. Let $\alpha(t) = (x(t), y(t))$ be a local parametrization of a plane curve C. Then the curvature of C at $\alpha(t)$ is given by ——————————.

(A) $\dfrac{x'y'' - x''y'}{\left(x'^2 + y'^2\right)^{3/2}}$.

(B) $\dfrac{x'y'' + x''y'}{\left(x'^2 - y'^2\right)^{3/2}}$.

(C) $\dfrac{x'y'' - x''y'}{\left(x'^2 + y'^2\right)^{2/3}}$.

(D) $\dfrac{x'y'' + x''y'}{\left(x'^2 - y'^2\right)^{2/3}}$.

20. The product, $f\omega$, of a function $f : U \to \mathbf{R}\left(U \text{ open in } \mathbf{R}^{n+1}\right)$ and a 1-form ω on U is defined by $(f\omega)(p, v) =$ ——————————.

(A) Not defined.

(B) $f(p, v)\,\omega(p, v)$.

(C) $f(p)\,\omega(p, v)$.

(D) $f(v)\,\omega(p, v)$.