D 143209 **(Pages : 3)** **Name.....**

Reg. No.....

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH4E11—GRAPH THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *the questions.*

Each question carries a weightage 1.

1. Distinguish between a trail and path. Illustrate with an example.
2. Define block and bridge of a simple graph. Illustrate with an example.
3. Detine a tree. Prove that a (p, q) graph G is a tree if and only if $p = q + 1$.
4. Define diameter of a connected graph. Illustrate with an example.
5. Define chromatic number χ (G) of a graph G. Prove that $\chi(K_n) = n$.
6. Prove that in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.
7. Define line independence number $\beta_1(G)$ of a graph G. Prove that $\beta_1(K_p) = \left[\frac{p}{2}\right]$.
8. Prove that in a k-critical graph $(k \geq 2)$, no vertex cut is a clique.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** *questions from each module.*

Each question carries a weightage 2.

MODULE I

9. Prove that in a tree any *two* vertices are connected by a path.

10. Prove that a graph G is a cut edge of G if and only if e contained no cycle of G.

11. Prove that the number of spanning trees of a complete graph K_n is n^{n-2}.

MODULE II

12. Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

13. Prove that for any graph G of order n, $\alpha + \beta = n$, where α is the independence number and β is the covering number of G.

14. Prove that every 3-regular graph without cut edges has a perfect matching.

MODULE III

15. If the graph G is k-critical, then prove that $\delta \geq k - 1$.

16. Prove that K_5 is non-planar.

17. State and prove Euler formula for a is a connected plane graph G.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*

Each question carries a weight of 5.

18. Prove that a non-empty connected graph is Eulerian if and only if it has no vertex of odd degree.

19. Explain the Chinese postman problem.

20. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge coloring in which both colors are represented at each vertex of degree at least two.

21. Prove that every planar graph is 5-vertex colourable.

(2 × 5 = 10 weightage)

D 143209–A **(Pages : 5)** **Name.....**

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MTH4E11—GRAPH THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH4E11—GRAPH THEORY

(Multiple Choice Questions for SDE Candidates)

1. What is the regularity of K_n.

   (A) n. (B) $n - 1$.

   (C) 0. (D) 1.

2. A connected planar graph having 6 vertices, 7 edges contains ———— regions.

   (A) 1. (B) 3.

   (C) 5. (D) 7.

3. Which of the following will be an upper bound for minimum degree of a graph with 10 vertices.

   (A) 9. (B) 8.

   (C) 7. (D) 6.

4. For a given graph G having v vertices and e edges which is connected and has no cycles, which of the following statements is true ?

   (A) $v = e$. (B) $v = e + 1$.

   (C) $v + 1 = e$. (D) $v = e - 1$.

5. A cycle on n vertices is isomorphic to its complement. The value of n is :

   (A) 3. (B) 4.

   (C) 5. (D) 6.

6. Which of the following statements is/are TRUE for Tree ?

   P : Every two points of G are joined by a unique path.

   Q : G is connected and $p = q - 1$.

   (A) P only. (B) Q only.

   (C) Both P and Q. (D) Neither P and Q.

7. Let κ (G) = vertex connectivity, λ (G) = edge connectivity and δ (G) = minimum degree of graph G. Then :

(A) $\kappa(G) \leq \delta(G) \leq \lambda(G)$.

(B) $\lambda(G) \leq \delta(G) \leq \kappa(G)$.

(C) $\delta(G) \leq \kappa(G) \leq \lambda(G)$.

(D) $\kappa(G) \leq \delta(G) \leq \lambda(G)$.

8. Let G = C_n. Then :

(A) There is path of length 5.

(B) There is a closed path of length 5.

(C) G is bipartite.

(D) None of the above.

9. Which of the following statement is/are TRUE ?

P : A cycle is a walk with end vertices are same.

Q : A cycle is a path with end vertices are same.

(A) P only.

(B) Q only.

(C) Both P and Q.

(D) Neither P and Q.

10. Number of edges incident with the vertex V is called?

(A) Degree of a graph.

(B) Handshaking lemma.

(C) Degree of a vertex.

(D) None of the above.

11. If a simple graph G, contains n vertices and m edges, the number of edges in the Graph G' (Complement of G) is ————.

(A) $\frac{n^2 + n - 2m}{2}$.

(B) $\frac{n^2 + 2n - 2m}{2}$.

(C) $\frac{n^2 - n - 2m}{2}$.

(D) $\frac{n^2 - 2n - 2m}{2}$.

12. The number of elements in the adjacency matrix of a graph having 7 vertices is ————————.

(A) 7. (B) 14.

(C) 36. (D) 49.

13. The graph in which, there is a closed trail which includes every edge of the graph is Known as ?

(A) Hamiltonian graph. (B) Euler graph.

(C) Directed graph. (D) Planar graph.

14. Let G be a simple graph. Which of the following statements is true ?

P : Adjacency matrix is symmetric.

Q : Trace of adjacency matrix is 1.

(A) P only. (B) Q only.

(C) Both P and Q. (D) Neither P and Q.

15. How many of the following statements are correct ?

1. All cyclic graphs are complete graphs.
2. All complete graphs are cyclic graphs.
3. All paths are bipartite.
4. All cyclic graphs are bipartite.
5. There are cyclic graphs which are complete.

(A) 1. (B) 2.

(C) 3. (D) 4.

16. Let G be a Null graph. Then :

(A) $V(G) = \phi$. (B) $E(G) = \phi$.

(C) $|E(G)|$ = Even. (D) $|E(G)|$ = Odd.

17. G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be :

(A) Regular. (B) Complete.

(C) Hamiltonian. (D) Euler.

18. Radius of a graph, denoted by rad (G) is defined by ————————?

(A) Max $\{e(v) : v \in V(G)\}$. (B) Min $\{e(v) : v \in V(G)\}$.

(C) Max $\{d(v, v) : v, u \in V(G)\}$. (D) Min $\{d(u, v) : v, u \in V(G)\}$.

19. If a graph is planar, then it is embeddable on a :

(A) Circle. (B) Square.

(C) Sphere. (D) Triangle.

20. A simple graph has ————————.

(A) Loops. (B) Parallel edges.

(C) Loops and parallel edges. (D) None of the above.