D 143210 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH 4E 12—REPRESENTATION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Let $A(x)$ be a representation of a group G and $x, y \in G$ be such that $y = txt^{-1}$ for some $t \in G$. Show that trace $A(x) =$ trace $A(y)$.

2. Let

$$A(x) = \begin{pmatrix} C(x) & 0 \\ E(x) & D(x) \end{pmatrix}$$

be a representation of a group G. Show that $D(x)$ is also a representation of G.

3. Let $N(x)$ be the permutation representation of the symmetric group S_3. Find $N(a)$ where $a = (12)$.

4. Let ν be the natural character of the symmetric group S_3. Find $\langle \nu, \nu \rangle$.

5. Let ρ be the character of the right regular representation of the cyclic group $G = \{1, a, a^2 : a^3 = 1\}$. Find $\rho(a)$.

6. Find the number of simple characters of the Klein Four group.

Turn over

7. Let G be the cyclic group $\mathbb{Z}_4$ and H be a subgroup of order 2. Let ϕ be the trivial character of H. Find $\phi^{G}(a)$ where $a \in G$.

8. Describe a simple character of degree 2 of the symmetric group S_3.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each module.*
Each question has weightage 2.

MODULE I

9. For each x in the symmetric group S_n let

$$\xi(x) = \begin{cases} 1 \text{ if } x \text{ is even} \\ -1 \text{ if } x \text{ is odd} \end{cases}.$$

Show that $\xi(x)$ is a representation of S_n.

10. Let V be a G-module. For each $x \in G$ let $\alpha_x : V \to V$ be defined by $v \mapsto vx$. Show that $x \mapsto \alpha_x$ is a representation of G by automorphisms of V.

11. Let $N(x)$ be the permutation representation of the symmetric group S_3. List all matrices $N(x)$ for $x \in S_3$.

MODULE II

12. Let ϕ be a character of a group G. Show that $\phi(x) = \phi(y)$ whenever x and y are conjugates in G.

13. Let $\chi^{(1)}, \chi^{(2)}, \ldots, \chi^{(k)}$ be all the distinct simple characters of a finite group G with degrees $f_1, f_2, \ldots, f_k$ respectively. Show that $\sum f_i^2 = g$ where g is the order of G.

14. Find α, β in the following character table of S_3.

	C_1	C_2	C_3
$\chi^{(1)}$	1	1	1
$\chi^{(2)}$	1	-1	1
$\chi^{(3)}$	2	α	β

Module III

15. Let $\pi(x)$ be a permutation representation of degree ≥ 2 of a finite group G. For each $x \in G$ let

$$\phi(x) = \text{number of symbols fixed by } \pi(x) - 1.$$

Show that ϕ is a character of G.

16. Let G be a transitive permutation group and H be the stabilizer of 1 and let $1p_\alpha = \alpha$ for each symbol α. Show that the stabilizer of α is $p_\alpha^{-1} H p_\alpha$.

17. Let G be a doubly transitive permutation group and ν be the natural character of G. Show that $\nu(x) - 1$ is a simple character of G.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Define irreducible G-module.

(b) Let $V = U_1 \oplus U_2$ be a G-module where U_1 and U_2 are submodules of V. Let $A(x)$ be a representation of G provided by the G-module V. Show that $A(x) \sim \text{diag}(A_1(x), A_2(x))$ where $A_1(x)$ is a representation corresponding to U_1 and $A_2(x)$ is a representation corresponding to U_2.

Turn over

19. (a) Define commutant algebra of a representation.

    (b) Show that if $A(x)$ is an irreducible representation of degree m over an algebraically closed field K and if T is an $m \times m$ matrix over K such that $A(x)\,T = T\,A(x)$ for all $x \in G$. Show that $T = \lambda I$ for some $\lambda \in K$.

20. (a) State the character relations of the second kind.

    (b) Show that for any character ϕ of a finite group G $\phi\left(x^{-1}\right) = \overline{\phi(x)}$.

    (c) Describe the character table of the cyclic group $\mathbb{Z}_3$.

21. Let G be a group, H be a subgroup of G and ϕ be a character of H. Let ϕ^G be the induced character of G. Then prove the following where the notations are the usual ones.

    (a) For each $x \in G, \phi^G(x) = \frac{1}{h}\sum_{y \in G} \phi\left(yxy^{-1}\right)$.

    (b) $\phi_\alpha^G = \frac{n}{h_\alpha}\sum_{z \in C_\alpha} \phi(z)$

    where C_α is a conjugacy class of G and $h_\alpha = |C_\alpha|$.

(2 × 5 = 10 weightage)

D 143210–A **(Pages : 6)** **Name.......................................**

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FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2026

(CBCSS)

Mathematics

MTH4E12—REPRESENTATION THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH4E12 REPRESENTATION THEORY

(Multiple Choice Questions for SDE Candidates)

1. Which of the following is true ?

   (A) A group G cannot have more than one representation by permutations.

   (B) A group G may have more than one representation by permutations, possibly of different degrees.

   (C) There exists a finite group G which does not have a right regular representation.

   (D) None of these.

2. Let A (x) be a matrix representation of a group G and let $\phi(x)$ be the character of A (x). Which of the following is true ?

   (A) Equivalent representations have the same character.

   (B) Equivalent representations need not have the same character.

   (C) There exists $x, y \in G$ with x and y conjugate in G and $\phi(x) \neq \phi(y)$.

   (D) None of these.

3. Let V = $[e_1, e_2, e_3]$ be a basis of $\mathbb{R}^3$ and let A denote the matrix representation of the linear map α on $\mathbb{R}^3$ defined by $\alpha(x, y, z) = (y, x + y, z)$. Then, A is :

   (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (B) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

   (D) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

4. If χ and $\overline{\chi}$ are the characters of A (x) and $A^{\dagger}(x)$ respectively, then :

   (A) χ is simple if and only if $\overline{\chi}$ is simple.

   (B) χ is simple but $\overline{\chi}$ need not be simple.

   (C) $\overline{\chi}$ is simple even when χ is not simple.

   (D) None of these.

5. Let G be a finite group with order g and let $\mathbb{K}$ be a field with characteristic 10. What is the sufficient condition on g which makes a reducible representation A (x) completely reducible ?

   (A) g divides 10. (B) $g = 10$.

   (C) g is prime to 10. (D) $g \neq 10$.

6. Suppose that U and V are irreducible G–modules over K. Then a G—homomorphism $\theta : V \to U$ is :

   (A) Always an isomorphism.

   (B) Always the zero map.

   (C) Neither zero nor an isomorphism.

   (D) Either zero or an isomorphism.

7. Let $\chi(x)$ and $\chi'(x)$ be the characters of the irreducible representations A (x) and B (x) respectively. Then, which of the following statements are true ?

   (i) $\langle \chi, \chi' \rangle = 0$ if $A(x) \sim B(x)$.

   (ii) $\langle \chi, \chi' \rangle = 1$ if $A(x) \sim B(x)$.

   (iii) $\langle \chi, \chi' \rangle = 0$ if $A(x) \nsim B(x)$.

   (A) Only (ii). (B) Both (ii) and (iii).

   (C) Both (i) and (iii). (D) Only (iii).

8. Two representations of a finite group over the complex field are equivalent if and only if :

   (A) They have the same character.

   (B) They have the same determinant.

   (C) They have the same character which is zero.

   (D) None of these.

9. Let D be a representation of the group S_3 of order 2 and let 1 be the identity in S_3. Then the character of D (1) is :

   (A) 1. (B) 2.

   (C) 0. (D) – 1.

10. Let R (x) = $r_{ij}(x)$ be the right-regular representation of G-module $G_{\mathbb{C}}$ with basis vectors $[x_1, x_2, ..., x_g]$. If $x_i x_j = x_{\mu}(i, j)$ in G, then $r_{ij(x_s)} =$

    (A) $\delta_{\mu(j,s),i}$. (B) $\delta_{\mu(j,s),j}$.

    (C) $\delta_{\mu(i,s),j}$. (D) $\delta_{\mu(i,s),i}$.

11. Let R (x) = $r_{ij}(x)$ be the right regular representation of a G-module $G_{\mathbb{C}}$ with basis vectors $[x_1 (= 1), x_2, ..., x_g]$ and let $\rho\ (x)$ be the character of R (x). Then, ρ is the (g-tuple).

    (A) $(g, 1, 1, 1, ..., 1)$. (B) $(0, 0, 0, 0, ..., g)$.

    (C) $(1, 1, 1, ..., 1, g)$ (D) $(g, 0, 0, 0, ..., 0)$.

12. Which of the following is true ?

    (A) Character may differ on a conjugacy class.

    (B) Character is constant on a conjugacy class and is the order of corresponding conjugacy class.

    (C) Character is constant on a conjugacy class and is always 1.

    (D) Character is constant on a conjugacy class.

13. Let G be a group of order g and let the number of elements in a conjugacy class $C_{\alpha}, (1 \le \alpha \le k)$ be denoted by h_{α}. Then, the equation $\sum_{\alpha=1}^{k} h_{\alpha} = g$ is called :

    (A) Character relation of first kind.
    (B) Class equation
    (C) Character relation of second kind.
    (D) None of these.

14. Which of the following is false ?

    (A) $\mathbb{Z}_3$ has precisely 3 characters.
    (B) All characters of $\mathbb{Z}_3$ are linear.
    (C) $\mathbb{Z}_3$ has 3 conjugacy classes.
    (D) $\mathbb{Z}_3$ has precisely 1 character, the trivial character.

15. Let A be a representation of G with character ϕ and let $N = \{u \in G : \phi(u) = \phi(1)\}$. Suppose p : N is a normal subgroup of G, q : N is the kernel of G. Then,

    (A) p and q are true.
    (B) p is true and q is false.
    (C) p is false and q is true.
    (D) p and q are false.

16. D_4 has :

    (A) 4! simple characters.
    (B) 8 simple characters.
    (C) 5 simple characters.
    (D) 4 simple characters.

17. Let G be a group with 3 simple characters $\chi^{(1)}, \chi^{(2)}, \chi^{(3)}$ and let $\chi^{(1)}(1) = 1, \chi^{(2)}(1) = 1, \chi^{(3)}(1) = 1$. Then, $|G|$ is :

    (A) 4.
    (B) 2.
    (C) 3.
    (D) 6.

18. Let G be a group with 4 simple characters $\chi^{(1)}, \chi^{(2)}, \chi^{(3)}, \chi^{(4)}$ and let $\chi^{(1)}(1) = 1, \chi^{(2)}(1) = 1$, . $\chi^{(3)}(1) = 1$, $\chi^{(4)}(1) = 3$,. Then, $|G|$ is :

    (A) 12. (B) 6.

    (C) 9. (D) 4.

19. If G has a permutation representation $\pi(x)$ of degree greater than unity, then the function $\phi(x) =$ (number of fixed points of $\pi(x)$) $- 1$ is a character of G that is :

    (A) Always simple. (B) Possibly compound.

    (C) Always compound. (D) None of these.

20. Let χ denote the permutation character of A_5. Then, χ ((123)) is :

    (A) – 1. (B) 2.

    (C) 3. (D) 1.