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FIFTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2026

MATHEMATICS

MAT5CJ302 Abstract Algebra I

2024 Admission Onwards

Maximum Time: 2 Hours	**Maximum Marks: 70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Let $*$ be a binary operation on $\mathbb{Q}$ defined by $a * b = ab + 1$. Determine whether $*$ is commutative and associative.
2	Prove that if $\phi : \langle S, * \rangle \longrightarrow \langle S', *' \rangle$ is an isomorphism, then $\phi^{-1} : \langle S', *' \rangle \longrightarrow \langle S, * \rangle$ is also an isomorphism.
3	Determine whether $H = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ is a subgroup of $(\mathbb{R}^2, +)$. Justify your answer.
4	Define a cyclic group and a generator of a group. Give an example of a cyclic group and list all its generators.
5	Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 2 & 8 & 1 & 4 & 7 & 3 \end{pmatrix}$.
6	Find all cosets of the subgroup $H = \{0, 4\}$ in $\mathbb{Z}_8$.
7	A finite group G has order 24. List all possible orders of a subgroup $H \leq G$.
8	Let $\phi : \mathbb{Z} \to \mathbb{Z}_{10}$ be defined by $\phi(n) = 6n$. Determine $\ker(\phi)$ and find $\phi(18)$.
9	Define an integral domain. Show that $\mathbb{Z}_p$ is an integral domain whenever p is prime.
10	Show that every finite integral domain is a field.

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Section B

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All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Let S be a finite set and let $*$ be a binary operation represented by a table. Suppose that there is an identity element and that every element occurs exactly once in each row and each column. Prove that every equation $a*x=b$ and $y*a=b$ has a unique solution in S.
12	Show that if G is a finite group with identity e and with an even number of elements, then there is $a \neq e$ in G such that $a*a=e$.
13	Let H be a subgroup of a group G. For $a, b \in G$, let $a \sim b$ if and only if $ab^{-1} \in H$. Show that $\sim$ is an equivalence relation on G.
14	Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and is the unique such element. Show that a is the unique element of G of order 2.
15	Show that if σ is a cycle of odd length, then σ^2 is a cycle.
16	Show that a group with at least two elements but with no proper nontrivial subgroups must be finite and of prime order.
17	Show that if G, G' and G'' are groups and if $\phi : G \to G'$ and $\gamma : G' \to G''$ are homomorphisms, then the composite map $\gamma\phi : G \to G''$ is a homomorphism.
18	Prove that:-In the ring $\mathbb{Z}_n$, the divisors of 0 are precisely those nonzero elements that are not relatively prime to n.

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	(a) Show that if $(a*b)^2 = a^2*b^2$ for a and b in a group G, then $a*b=b*a$. (b) Let G be a group and let g be one fixed element of G. Show that the map i_g, such that $i_g(x) = gxg'$ for $x \in G$, is an isomorphism of G with itself.
20	(a) Let A be a nonempty set, and let S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication. (b) Let a and b be elements of a group G. Show that if ab has finite order n, then ba also has order n.