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FIFTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2026

MATHEMATICS

MAT5CJ303 Complex Analysis I

2024 Admission Onwards

Maximum Time: 2 Hours	**Maximum Marks: 70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	The three points $z_1 = 1+5i$, $z_2 = -4-i$, $z_3 = 3+i$ are vertices of a triangle. Find the length of the median from z_1 to the side joining z_2 and z_3.
2	Using de Moivre's formula, find $\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6$.
3	Find the natural domain and range of the complex function $f(z) = \frac{z}{\lvert z\rvert}$. Give a suitable justification for your answer.
4	Find the image of the line $y = 1$ under the complex mapping $w = z^2$.
5	Find the image of the region $\lvert z\rvert \leq 8$, $\frac{\pi}{2} \leq \arg(z) \leq \frac{3\pi}{4}$ under the principal square-root mapping $w = z^{1/2}$.
6	Show that the function $f(z) = z^2 - iz + 3 - 2i$ is continuous at $z_0 = 2 - i$.
7	Let $f(z) = \lvert z\rvert^2$. Show that f is differentiable at the origin but is not differentiable at any point $z \neq 0$.
8	Find all complex values of $\ln(1+i)$.
9	Verify the identity $\cos^2 z + \sin^2 z = 1$.
10	Express $\sinh\left(\frac{\pi}{2}i\right)$ in the form $a+ib$.

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Solve the equation $z^2 - 2z + 10 = 0$ and express its roots in the form $a + ib$.
12	Find the fifth roots of unity and represent them in polar form. Also state the geometrical figure formed by the roots in the complex plane.
13	Find the image of the line $y = 2x + 1$ under the complex mapping $w = (1 - i)z$.
14	Using the principal square-root function, find $z^{1/2}$ at $z = -i$.
15	Let f be a complex-valued function defined on a domain D. Prove that if $\lim_{z \to z_0} f(z) = L$, then $\lim_{z \to z_0} \lvert f(z) \rvert = \lvert L \rvert$. Hence use the result to evaluate $\lim_{z \to 1+i} \lvert z^2 + 2z \rvert$.
16	Verify that $u(x, y) = x^2 - y^2$ is harmonic. Find a harmonic conjugate $v(x, y)$ and form the corresponding analytic function $f(z) = u + iv$.
17	Consider the exponential mapping $w = e^z, \quad z = x + iy$. Show that $w = e^x(\cos y + i \sin y)$. Hence, determine the image of the horizontal line $y = b$ under the mapping $w = e^z$, and identify the resulting curve.
18	Solve the complex equation $\cos z = 4$ and find all complex values of z.

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	(a) Find an upper bound for $\left\lvert \dfrac{-1}{z^4 + 3z^2 + 2} \right\rvert$ if $\lvert z \rvert = 2$. (b) Prove that, for any two complex numbers z_1, z_2, $\lvert \lvert z_1 \rvert - \lvert z_2 \rvert \rvert \leq \lvert z_1 - z_2 \rvert$.
20	Verify that $u(r, \theta) = r^3 \cos 3\theta$ is harmonic in a domain not containing the origin. Using the Cauchy–Riemann equations in polar coordinates, find a harmonic conjugate $v(r, \theta)$ and form the corresponding analytic function $f(z) = u + iv$.