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FIFTH SEMESTER (CUFYUGP) DEGREE EXAMINATION NOVEMBER 2026

MATHEMATICS

MAT5EJ303 (2) Convex Optimization

2024 Admission Onwards

Maximum Time: 2 Hours	**Maximum Marks: 70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Find the directional derivative of $f(x, y) = x^2 + xy + y^2$ at the point $(1, 2)$ in the direction of the vector $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$.
2	Find the derivative matrix (Jacobian matrix) of the function $\mathbf{f}(x, y) = \begin{pmatrix} x^2 + y \\ xy + y^2 \end{pmatrix}$ at the point $(1, 2)$.
3	Determine whether $f(x, y) = x^2 + y^2$ has a local maximum, local minimum, or saddle point at $(0, 0)$.
4	Define an affine set and a convex set. State the condition that distinguishes an affine combination from a convex combination.
5	Distinguish between convex, strictly convex, concave, and strictly concave functions.
6	State the three conditions that must be satisfied for an optimization problem to be a convex optimization problem in standard form.
7	Explain how a quasiconvex optimization problem can be solved using a sequence of convex feasibility problems.
8	Define a monomial and a posynomial function. Give one example of each.
9	Consider the standard-form linear program:- **Minimize** $c^T x$ subject to $Ax = b, \quad x \geq 0$. Write its Lagrangian and state the corresponding dual problem.
10	For the convex optimization problem:- **Minimize** $\frac{1}{2}x^T Px + q^T x$ subject to $Ax = b$, write the KKT conditions in matrix form.

Section B

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All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Let $f(x, y) = x^2y + x^3 - 2xy$. Find the directional derivative of f at the point $(1, 2)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$. Hence, find the direction in which f increases most rapidly at $(1, 2)$.
12	Let $w = x^2y + yz^2$, where $x = r + s, \quad y = r - s, \quad z = rs$. Using the chain rule, find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$.
13	Prove that a set $C \subseteq \mathbf{R}^n$ is convex if and only if its intersection with every line is convex. Also prove the corresponding result for affine sets.
14	Consider the quadratic function $f(x) = \frac{1}{2}x^TPx + q^Tx + r, \quad P = P^T$. Show that $\nabla^2 f(x) = P$. Hence give necessary and sufficient conditions on P for f to be convex, strictly convex, concave, and strictly concave.
15	Consider the quadratic function $f_0(x) = \frac{1}{2}x^TPx + q^Tx + r$. Show that the necessary and sufficient condition for $x^\star$ to minimize f_0 is $Px^\star + q = 0$. Discuss the cases when $P \succ 0$ and when P is singular.
16	Consider the semidefinite program:- **Minimize** c^Tx subject to $x_1F_1 + \cdots + x_nF_n + G \succeq 0, \quad Ax = b$. Explain the meaning of the positive semidefinite constraint. Show that if $G, F_1, \ldots, F_n$ are diagonal matrices, the semidefinite program reduces to a linear program.
17	Consider the optimization problem:- **Minimize** $x^2 + 1$ subject to $(x - 2)(x - 4) \leq 0$. Find the feasible set, the optimal solution, and the optimal value. Hence, form the Lagrangian and obtain the Lagrange dual problem.
18	Consider the equality-constrained convex quadratic problem:- **Minimize** $\frac{1}{2}x^TPx + q^Tx + r$ subject to $Ax = b, \quad P \succeq 0$. (a) Form the Lagrangian. (b) Derive the KKT conditions. (c) Write the KKT conditions as a block matrix system. (d) Explain how solving this system gives the primal and dual optimal solutions.

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19 Show that the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x_1, x_2) = \begin{cases} \dfrac{x_1^3 x_2}{x_1^2 + x_2^2}, & (x_1, x_2) \neq (0, 0), \\ 0, & (x_1, x_2) = (0, 0) \end{cases}$$

has a continuous first derivative $f' : \mathbb{R}^2 \to \mathbb{R}^2$ at $(0, 0)$ and that all its second order partial derivatives exist at $(0, 0)$. Prove that

$$f_{12}(0, 0) \neq f_{21}(0, 0).$$

20 Let $f(x, y) = \frac{x^2}{y}, \qquad y > 0.$

(a) Find the Hessian matrix of f.

(b) Determine whether the Hessian is positive semidefinite.

(c) Hence establish the convexity of f on $\mathbf{R} \times \mathbf{R}_{++}$.