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		Register No.

FIFTH SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2026

MATHEMATICS

MAT5CJ301 Real Analysis II

2024 Admission Onwards

Maximum Time: 2 Hours	**Maximum Marks: 70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling: 24 Marks)

1	Using the definition of continuity, prove that $h(x) := x^2$ is continuous on $\mathbb{R}$.
2	Give an example of functions f and g that are both discontinuous at a point c in $\mathbb{R}$ such that (a) the sum $f+g$ is continuous at c (b) the product fg is continuous at c.
3	Using the definition, find the derivative of $g(x) = \frac{1}{x}; x \in \mathbb{R}, x \neq 0$
4	Show that $f : \mathbb{R} \mapsto \mathbb{R}$ defined by $f(x) = \begin{cases} x^2 & \text{if } x \text{ is irrational} \\ x & \text{if } x \text{ is rational} \end{cases}$ is not differentiable at 0
5	Define a partition of a closed bounded interval $[a, b]$. What is meant by the norm (mesh) of a partition?
6	If $f, g \in \mathcal{R}[a, b]$, state whether the product fg is Riemann integrable on $[a, b]$.
7	Let $f \in \mathcal{R}[a, b]$. State the condition under which $\int_a^b f(x)\,dx = 0$ is guaranteed when $f(x) \geq 0$ on $[a, b]$.
8	Show that $\lim \left(\frac{x}{x+n} \right) = 0$ for all $x \in \mathbb{R}, x \geq 0$
9	Show that $(f_n(x)) = (x^n)$ does not converge uniformly to $f = \mathbf{0}$ on $(-1, 1)$
10	Explain Pointwise Convergence

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling: 36 Marks)

11	Prove that:- A function $f : A \mapsto \mathbb{R}$ is continuous at the point $c \in A$ if and only if for every sequence (x_n) in A that converges to c, the sequence $(f(x_n))$ converges to $f(c)$.
12	State and prove Bolzano's Intermediate Value Theorem
13	Prove that:- Let c be an interior point of the interval I at which $f : I \mapsto \mathbb{R}$ has a relative extremum. If the derivative of f at c exists, then $$f'(c) = 0$$
14	Suppose that f is continuous on the closed interval $I := [a; b]$, that f is differentiable on the open interval (a, b), and that $f'(x) = 0$ for $x \in (a, b)$. Then prove that f is constant on I.
15	Solve $\int_1^4 \frac{\cos\sqrt{t}}{\sqrt{t}}\, dt$
16	Prove that every monotone function on $[a, b]$ is Riemann integrable.
17	Prove that:-A sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\| \longrightarrow 0$
18	Evaluate $\lim\left(\frac{x^n}{1+x^n}\right)$ for all $x \in \mathbb{R}, x \geq 0$

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks)

19	(a) If $f(x) := \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$, show that f is discontinuous everywhere. (b) Show that if f and g are uniformly continuous on a subset A of R, then $f + g$ is uniformly continuous on A.

20	Let f be continuous on the interval $I = [a; b]$ and let c be an interior point of I. Assume that f is differentiable on (a, c) and (c, b) . Then prove that (a) If there is a neighborhood $(c - \delta, c + \delta) \subseteq I$ such that $f'(x) \geq 0$ for $c - \delta < x < c$ and $f'(x) \leq 0$ for $c < x < c + \delta$ then f has a relative maximum at c. (b) If there is a neighborhood $(c - \delta, c + \delta) \subseteq I$ such that $f'(x) \leq 0$ for $c - \delta < x < c$ and $f'(x) \geq 0$ for $c < x < c + \delta$ then f has a relative minimum at c.

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