D 150346 **(Pages : 4)** **Name.....**

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE (CBCSS) REGULAR / SUPPLEMENTARY EXAMINATION, NOVEMBER 2026

Mathematics

MTH3C14—PDE AND INTEGRAL EQUATIONS

(2019 Admission Onwards)

Time : Three Hours Maximum : 30 Weightage

Part A – Short Answer Type Questions

Answer **all** *questions.*

Each question carries a weightage 1.

1. Give an example of a nonlinear first-order PDE.
2. When is a second-order linear PDE in two independent variables classified as elliptic ?
3. Define the term Eikonal Equation.
4. Let $u(x, t)$ be a solution of the wave equation :

   $u_{tt} - c^2 u_{xx} = 0,$

   which is defined in the whole plane ? Assume that u is constant on the line $x = 2 + ct$. Prove that

   $u_t + cu_x = 0.$
5. Define Dirichlet problem.
6. Define Volterra equation. Give an example.
7. Discuss briefly: the Green's function.
8. Define problem of the third kind (Robin problem).

(8 × 1 = 8 weightage)

Turn over

Part B - Paragraph Type Questions

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. Solve the first-order linear PDE $u_x + 3u_y = 0$ with the initial condition $u(x,0) = \sin x$.

10. Classify and reduce the equation $u_{xx} - 4u_{xy} + 4u_{yy} = 0$ to its canonical form.

11. Consider the Tricomi equation:

    $u_{xx} + xu_{yy} = 0$ for $x < 0$.

    Find a mapping $q = q(x,y), r = r(x,y)$ that transforms the equation into its canonical form, and present the equation in this coordinate system.

Module II

12. Solve the equation

    $u_t = 17u_{xx}, 0 < x < \pi, t > 0$,

    with the boundary conditions

    $u(0,t) = u(\pi,t) = 0, t \geq 0$,

    and the initial conditions

$$u(x,0) = \begin{cases} 0, 0 \leq x \leq \dfrac{\pi}{2}, \\ 2, \dfrac{\pi}{2} < x \leq \pi \end{cases}$$

13. Explain the steps for applying the separation of variables method to the wave equation.

14. Prove that a necessary condition for the existence of a solution to the Neumann problem is :

    $\int_{\partial D} g(x(s), y(s))ds = \int_D F(x,y)dx\,dy$

    where $(x(s), y(s))$ is a parameterization of ∂D.

Module III

15. Prove that :

$$\int_a^x \cdots \int_a^x f(x)\, \overbrace{dx \cdots dx}^{n \text{ times}} = \overbrace{x \cdots x}^{n \text{ times}} \frac{1}{(n-1)!}\int_a^x (x-\xi)^{n-1} f(\xi)\, d\xi .$$

16. Transform :

$$\left.\begin{aligned} &\frac{d^2y}{dx^2} + \lambda y = f(x) \\ &y(0) = 1, y'(0) = 0 \end{aligned}\right\}$$

to an integral equation.

17. Transform the problem :

$$y'' + xy = 1, y(0) = 0, y(l) = 1$$

to an integral equation.

(6 × 2 = 12 weightage)

Part C - Essay Type Questions

Answer **two** *questions.*
Each question carries a weightage 5.

18. Apply the method of characteristics to solve the general nonlinear first-order equation $F(u_x, u_y, u, x, y) = 0$.

19. Consider the Dirichlet problem in a bounded domain :

$$\Delta u = f(x,y), (x,y) \in D,$$
$$u(x,y) = g(x,y), (x,y) \in \partial D$$

Then prove that the problem has at most one solution in $C^2(D) \cap C(\bar{D})$.

Turn over

20. Derive and explain the significance of the Green's identities for elliptic problems.

21. Prove that the relation :

$$y(x) = \int_a^b G(x,\xi)\,\Phi(\xi)\,d\varsigma$$

where

$$G(x,\xi) = \begin{cases} -\frac{1}{A}u(x)v(\xi) \text{ when } x < \xi, \\ -\frac{1}{A}u(\xi)v(x) \text{ when } x > \xi, \end{cases}$$

implies the differential equation :

$$. L_y + \Phi(\xi) = 0$$

(2 × 5 = 10 weightage)