D 150350 **(Pages : 4)** **Name.....**

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2026

(CBCSS)

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define the probability mass function (pmf) for a discrete random variable X.
2. Two players, A and B, play a coin-tossing game. A gives B one dollar if a head turns up; otherwise, B pays A one dollar. If the probability that the coin shows a head is p, find the expected gain of A.
3. A box contains good and defective items. If an item drawn is good, we assign the number 1 to the drawing; otherwise, the number 0. Let p be the probability of drawing at random a good item. Then find $\mathrm{F}(x) = \mathrm{P}\{\mathrm{X} \le x\}$.
4. Define DF of the $\mathrm{RV}(\mathrm{X}, \mathrm{Y})$
5. When RVs X and Y are identically distributed ?
6. Define covariance between X and Y and give the equation for the evaluation of the same.
7. Define converges in the r^{th} mean.
8. When we say that sequence obeys Strong Law of Large Numbers (SLLN) ?

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module*
Each question carries a weightage 2.

MODULE I

9. Prove that the RV X defined on the probability space $(\Omega, \mathcal{S}, \mathrm{P})$ induces a probability space $(\mathcal{R}, \mathfrak{B}, \mathrm{Q})$ by means of the correspondence

$$\mathrm{Q}(\mathrm{B}) = \mathrm{P}\left\{\mathrm{X}^{-1}(\mathrm{B})\right\} = \mathrm{P}\left\{w : \mathrm{X}(w) \in \mathrm{B}\right\} \quad \text{for all } \mathrm{B} \in \mathfrak{B}.$$

10. Prove that every DF is the DF of an RV on some probability space.

11. Let X be an RV on a probability space $(\Omega, \mathcal{S}, \mathrm{P})$. Let $\mathrm{E}\,|\,\mathrm{X}\,|^k < \infty$ for some $k > 0$. Then prove that

$$n^k\, \mathrm{P}\left\{|\,\mathrm{X}\,| > n\right\} \to 0 \text{ as } n \to \infty.$$

MODULE II

12. If $\mathrm{X}_1, \mathrm{X}_2, ..., \mathrm{X}_n$ are independent, then prove that every subcollection $\mathrm{X}_{i_1}, \mathrm{X}_{i_2}, ..., \mathrm{X}_{i_k}$ of $\mathrm{X}_1, \mathrm{X}_2, ..., \mathrm{X}_n$ is also independent.

13. Prove the following :

    (a) The correlation co-efficient ρ between two RVs X and Y satisfies

    $$|\,\rho\,| \le 1.$$

    (b) The equality $|\,\rho\,| = 1$ holds if and only if there exist constants $a \neq 0$ and b such that $\mathrm{P}(a\mathrm{X} + b = \mathrm{Y}) = 1$.

14. If $\mathrm{EX}^2 < \infty$, then prove that

$$\mathrm{var}(\mathrm{X}) = \mathrm{var}\left(\mathrm{E}\left[\mathrm{X}|\mathrm{Y}\right]\right) + \mathrm{E}\left[\mathrm{var}\left(\mathrm{X}|\mathrm{Y}\right)\right].$$

MODULE III

15. If $\{A_n\}$ is an independent sequence of events such that $\sum_{n=1}^{\infty} P\,A_n = \infty$, then prove that $P\,A = 1$.

16. If $\sum_{n=1}^{\infty} \text{var}(X_n) < \infty$, then prove that $\sum_{n=1}^{\infty} (X_n - EX_n)$ converges almost surely

17. Let $X_1, X_2, ..., X_n$ be iid RVs such that $n^{-1/2} S_n$ has the same distribution for every $n = 1, 2, ...$ Then, prove that if $EX_i = 0, \text{var}(X_i) = 1$, the distribution of X_i must be $N(0, 1)$.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer **two** *questions.*
Each question carries a weightage 5.

18. Prove that the function F defined in

$$F(x) = Q(-\infty, x] = P\{w : X(w) \le x\} \text{ for all } x \in \mathcal{R}.$$

is a DF.

19. Let $\beta_n = E|X|^n < \infty$. Then prove that for arbitrary $k, 2 \le k \le n$,

$$\beta_{k-1}^{1/(k-1)} \le \beta_k^{1/k}.$$

20. Prove that the joint PDF of $X_{(j)}$ and $X_{(k)}$ is given by

$$g_{jk}(y_j, y_k) =$$

$$\begin{cases} \dfrac{n!}{(j-1)!(k-j-1)!(n-k)!} F^{j-1}(y_j)\left[F(y_k) - F(y_j)\right]^{k-j-1}\left[1 - F(y_k)\right]^{n-k} f(y_j) f(y_k) & \text{if } y_j < y_k, \\ 0 & \text{otherwise.} \end{cases}$$

Turn over

21. Let $\{X_n\}$ be a sequence of iid RVs with $0 < \text{var}(X_n) = \sigma^2 < \infty$ and common mean μ. Let $S_n = \sum_{j=1}^{n} X_j$, $n = 1, 2, \ldots$. Then prove that for every $x \in \mathbb{R}$,

$$\lim_{n \to \infty} P\left\{\frac{S_n - n\mu}{\sigma\sqrt{n}} \leq x\right\} = \lim_{n \to \infty} P\left\{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq x\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du. \quad (1)$$

(2 × 5 = 10 weightage)

D 150350-A **(Pages : 4)** **Name.....**

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THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2026

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 3E 04—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. A continuous variable can take any values within its ————.

   (A) Range. (B) Co domain.

   (C) Domain. (D) All of the above.

2. The random variables X and Y have variance 0.2 and 0.5 respectively. Let Z = 5X – 2Y. The variance of Z is :

   (A) 3. (B) 7.

   (C) 4. (D) 5.

3. If X is a random variable having its p.d.f.

   $f(x)$, the E(X) is called :

   (A) Arithmetic mean. (B) Geometric mean.

   (C) Harmonic mean. (D) First quartile.

4. Two random variables X and Y are said to be independent if :

   (A) E(XY) = 1. (B) E(XY) = 0.

   (C) E(XY) = E(X)E(Y). (D) E(XY) = any constant value.

5. If X and Y two independent variables and their expected values are $\bar{X}$ and $\bar{Y}$ respectively :

   (A) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=0.$

   (B) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=1.$

   (C) $E\left\{\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)\right\}=C$ (constant).

   (D) All the above.

6. If X is a random variable and its p.d.f. is $f(x)$, E(log x) represents :

   (A) Arithmetic mean. (B) Geometric mean.

   (C) Harmonic mean. (D) Logarithmic mean.

7. For two r.v.s X and Y, the relation E(XY) = E(X) E(Y) holds good :

   (A) If X and Y are independent. (B) For all X and Y.

   (C) If X and Y are identical. (D) If X and Y are discrete.

8. The correlation coefficient ρ between two r.vs satisfies :

(A) $|\rho| = 1$. (B) $|\rho| \leq 1$.

(C) $|\rho| < 0$. (D) $\rho > 1$.

9. Let (X, Y) be an RV with distribution function F. Then the marginal distribution function of X is ————.

(A) $F_1(x) = \lim_{x \to \infty} F(x, y)$. (B) $F_1(x) = \lim_{y \to \infty} F(x, y)$.

(C) $F_1(x) = \lim_{y \to 0} F(x, y)$. (D) $F_1(x) = \lim_{x \to 0} F(x, y)$.

10. Random variables X and Y are independent if and only if ————.

(A) $F(x, y) = F_1(x) + F_2(y)$ for all $(x, y) \in R_2$.

(B) $F(x, y) = F_1(x) - F_2(y)$ for all $(x, y) \in R_2$.

(C) $F(x, y) = F_1(x) / F_2(y)$ for all $(x, y) \in R_2$.

(D) $F(x, y) = F_1(x) * F_2(y)$ for all $(x, y) \in R_2$.

11. If $f(x)$ is a probability density function of a continuous random variable then ————.

(A) $\int_{-\infty}^{\infty} f(x)dx = 0$. (B) $\int_{-\infty}^{\infty} f(x)dx = 1$.

(C) $\int_{-\infty}^{\infty} f(x)dx > 1$. (D) $\int_{-\infty}^{\infty} f(x)dx < 0$.

12. The second moment about the mean is ————.

(A) Variance. (B) Mean.

(C) Standard deviation. (D) None of these.

13. The law of large numbers deals with what ?

(A) Running for a long time. (B) Probability.

(C) Geometry shapes. (D) Algebraic equations.

14. The law of large numbers shows a relationship between the theoretical probability and the ————.

(A) Rational probability. (B) Experimental probability.

(C) Exponential probability. (D) Sample size.

15. Let $X_1, X_2, \ldots$ be a sequence of independent, identically distributed random variables, each having finite mean $\mu = E(X_k), k = 1, 2, \ldots$. Then for any $\varepsilon > 0$, which of the following options is/are correct according to statement of weak law of large numbers.

(A) $P\left[\lim_{n\to\infty} \frac{X_1 + X_2 + \ldots + X_n}{n} = \mu\right] = 1.$

(B) $\lim_{n\to\infty} P\left[\left|\frac{X_1 + X_2^n + \ldots + X_n}{n} = \mu\right| \geq \varepsilon\right] = 0.$

(C) None of the above.

(D) Question does not provide sufficient data or is vague.

16. Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then :

(A) $X_n + Y_n \xrightarrow{L} X + c.$ (B) $X_n + Y_n \xrightarrow{L} X + Y.$

(C) $X_n + Y_n \xrightarrow{L} X.$ (D) $X_n + Y_n \xrightarrow{L} XY.$

17. Which of the statement(s) is/are false ?

(A) Convergence in probability closed under addition and multiplication.

(B) Convergence in probability closed under addition and division.

(C) Convergence in probability closed under addition and subtraction.

(D) All the above.

18. Suppose X is some r.v. then $X_m - X_n \xrightarrow{P} 0 \Leftrightarrow$:

(A) $X_m \xrightarrow{P} X.$ (B) $X_m \xrightarrow{P} 0.$

(C) $X_n \xrightarrow{P} 0.$ (D) $X_n \xrightarrow{P} X.$

19. Suppose X ~ P(λ). What is the distribution of $Y = \frac{(X - \lambda)}{\sqrt{\lambda}}$?

(A) N(λ, σ). (B) N(0, 1).

(C) N(μ, 1). (D) N(λ, 1).

20. Srong law of large numebrs indicates :

(A) Convergence in probability. (B) Convergence in distribution.

(C) Almost sure convergence. (D) None of the above.