D 150344 **(Pages : 4)** **Name.....**

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2026

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. If $\sum_{n=0}^{\infty} a_n$ converges absolutely then prove that $\sum_{n=0}^{\infty} a_n$ an converges.

2. If S is a Möbius transformation then prove that S is the composition of translations, dilations, and the inversion.

3. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n^p} z^n$.

4. Let γ be a rectifiable curve in $\mathbb{C}$ and suppose that F_n and F are continuous functions on $\{\gamma\}$. If

$$F = u - \lim F_n$$

on $\{\gamma\}$ then prove that

$$\int_{\gamma} \lim F_n = \lim \int_{\gamma} F_n.$$

Turn over

5. If $\gamma:[0,1]\to\mathbb{C}$ is a closed rectifiable curve and $a\notin\{\gamma\}$ then prove that

$$\frac{1}{2\pi i}\int_{\gamma}\frac{dz}{z-a}$$

is an integer.

6. Evaluate $\int_{\gamma}\frac{2z}{z^2+z+1}dz$ where γ is the circle $|z|=2$.

7. Prove that a function $f:[a,b]\to\mathbb{R}$ is convex if and only if the set

$$A=\{(x,y):a\le x\le b \text{ and } f(x)\le y\}$$

is convex.

8. State Hadamard's Three Circle Theorem.

(8 × 1 = 8 weightage)

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module*
Each question carries a weightage 2.

Module I

9. Let f and g be analytic on G and Ω respectively and suppose $f(G)\subset\Omega$. Then prove that $g\circ f$ is analytic on G and

$$(g\circ f)'=g'(f(z))f'(z)$$

for all z in G.

10. Let G and Ω be open subsets of $\mathbb{C}$. Suppose that $f:G\to\mathbb{C}$ and $g:\Omega\to\mathbb{C}$ are continuous functions such that $f(G)\subset\Omega$ and $g(f(z))=z$ for all z in G. If g is differentiable and $g'(z)\neq 0$, then prove that f is differentiable and

$$f'(z)=\frac{1}{g'(f(z))}.$$

11. If $\gamma : [a, b] \to \mathbb{C}$ is piecewise smooth then prove that γ is of bounded variation and

$$V(\gamma) = \int_a^b |\gamma'(t)| \, dt.$$

MODULE II

12. State and prove the Fundamental Theorem of Algebra.

13. If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point $a \in G$ with

$$|f(a)| \geq |f(z)| \text{ for all } z \in G,$$

then prove that f is constant.

14. Let G be simply connected and let $f : G \to \mathbb{C}$ be an analytic function such that $f(z) \neq 0$ for any z in G. Then prove that there is an analytic function $g : G \to \mathbb{C}$ such that $f(z) = \exp g(z)$.

MODULE III

15. Let $z = a$ be an isolated singularity of f and let

$$f(z) = \sum_{-\infty}^{\infty} a_n (z - a)^n$$

be its Laurent Expansion in ann $(a; 0, R)$. Then prove that $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \leq -1$.

16. If f has an isolated singularity at a then prove that $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z - a) f(z) = 0$.

17. Show that

$$\int_0^{\infty} \frac{\sin x}{x} \, dx = \frac{\pi}{2}.$$

(6 × 2 = 12 weightage)

Turn over

Part C (Essay Type Questions)

Answer **two** *questions.*
Each question carries a weightage 5.

18. Let G be either the whole plane $\mathbb{C}$ or some open disk. If $u : G \to \mathbb{R}$ is a harmonic function then prove that u has a harmonic conjugate.

19. Let G be an open subset of the plane and $f : G \to C$ an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma ; w) = 0$ for all $w \in C \sim G$, then prove that for $a \in G \sim \{\gamma\}$,

$$n(\gamma ; a) f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{z - a} dz.$$

20. State and prove Goursat's Theorem.

21. (a) State and prove Gasorati-Weierstrass theorem.

    (b) Let f be analytic in the region G except for the isolated singularities $a_1, a_2, ..., a_m$. If γ is a closed rectifiable curve in G which does not pass through any of the points a_k and if $\gamma \approx 0$ in G. Then prove that

$$\frac{1}{2\pi i} \int_{\gamma} f = \sum_{k=1}^{m} n(\gamma ; a_k) \operatorname{Res}(f, a_k).$$

(2 × 5 = 10 weightage)

D 150344-A **(Pages : 6)** **Name.....**

Reg. No.....

THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2026

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 3C 12—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. The unit sphere in $\mathbb{R}^3$ is given by ————.

   (A) $\mathcal{S} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\right\}$.

   (B) $\mathcal{S} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 \leq 1\right\}$.

   (C) $\mathcal{S} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 = 1\right\}$.

   (D) None of the above.

2. If $\sum_{n=0}^{\infty} a_n$ converges absolutely then :

   (A) $\sum_{n=0}^{\infty} a_n$ converges to a_0. (B) $\sum_{n=0}^{\infty} a_n$ converges to 1.

   (C) $\sum_{n=0}^{\infty} a_n$ converges to 0. (D) $\sum_{n=0}^{\infty} a_n$ converges.

3. Which one of the following is exponential series ?

   (A) $\sum_{n=0}^{\infty} \frac{z^n}{n+1!}$. (B) $\sum_{n=0}^{\infty} \frac{z^{n-1}}{n!}$.

   (C) $\sum_{n=0}^{\infty} \frac{z^{n+1}}{n!}$. (D) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

4. A function $f : G \to \mathbb{C}$ is analytic if ————.

   (A) f is continuous on G.

   (B) f is continuously differentiable on G.

   (C) f is differentiable at some points on G.

   (D) None of the above options.

5. If $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ then a_n = ————.

   (A) $\frac{1}{(n+1)!} f^{(n)}(a)$. (B) $\frac{1}{n!} f^{(n)}(a)$.

   (C) $f^{(n)}(a)$ (D) None of the above options.

6. Let $f : G \to \mathbb{C}$ be analytic and let

$$u(x,y) = \mathrm{Re} f(x+iy),\ v(x,y) = \mathrm{Im} f(x+iy) \text{ for } x+iy \text{ in G.}$$

Then ————.

(A) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.

(B) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

(C) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

(D) None of the above options.

7. Consider the function defined by $f(z) = z^2$. If $z = x + iy$ and $\mu + iv = f(z)$ then the hyperbolas

$$x^2 - y^2 = c \text{ and } 2xy = d$$

are mapped f into ————.

(A) the straight lines $\mu = c$, $v = d$.

(B) the straight lines $\mu = 1$, $v = 1$.

(C) the straight lines $\mu = d$, $v = c$.

(D) the unit circle.

8. Which one of the following is a definition of smooth path in a region :

(A) If $\gamma : [a, b] \to G$ is such that $\gamma'(t)$ exists for each t in $[a, b]$ and $\gamma' : [a, b] \to \mathbb{C}$ is continuous then γ is a smooth path.

(B) If $\gamma : [a, b] \to G$ is such that $\gamma'(t)$ exists for each t then γ is a smooth path.

(C) If $\gamma : [a, b] \to G$ is such that $\gamma'(t) = 0$ for each t in $[a, b]$ and γ is a smooth path.

(D) None of the above options.

9. Which one of the following statements is false :

(A) $f(z) = e^z$ is conformal throughout $\mathbb{C}$.

(B) $f(z) = e^z$ is continuous throughout $\mathbb{C}$.

(C) $f(z) = e^z$ is differentiable throughout $\mathbb{C}$.

(D) $f(z) = e^z$ is not conformal throughout $\mathbb{C}$.

10. Which one of the following is a dilation ?

(A) $S(z) = z + a$ for some complex number a.

(B) $S(z) = az$ for some complex number $a \neq 0$.

(C) $S(z) = 3z$.

(D) None of the above options.

Turn over

11. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $\mathcal{S} : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by

$$\mathcal{S}(z) = \frac{z - z_3}{z - z_4} \text{ if } z_2 = \infty.$$

Then $\mathcal{S}(z_2) =$ ————.

(A) 1. (B) 0.

(C) -1. (D) None of the above options.

12. Let z_1, z_2, z_3, z_4 be points in $\mathbb{C}_\infty$. Define $\mathcal{S} : \mathbb{C}_\infty \to \mathbb{C}_\infty$ by

$$\mathcal{S}(z) = \frac{z - z_3}{z - z_4} \text{ if } z_2 = \infty.$$

Then $\mathcal{S}(z_4) =$ ————.

(A) 1. (B) 0.

(C) ∞. (D) None of the above options.

13. If $\gamma : [a, b] \to \mathbb{C}$ be of bounded variation. Then if P and Q are partitions of $[a, b]$ and $P \subset Q$ then ————.

(A) $v(\gamma; P) = v(\gamma; Q)$. (B) $v(\gamma; P) \leq v(\gamma; Q)$.

(C) $v(\gamma; P) \geq v(\gamma; Q)$. (D) $v(\gamma; P) - v(\gamma; Q) = 1$.

14. Let f and g be continuous functions on $[a, b]$ and let γ and σ be functions of bounded variation on $[a, b]$. Then for any scalars α and β, ————.

(A) $\int_a^b (\alpha f + \beta g) d\gamma = |\alpha| \int_a^b f \, d\gamma - |\beta| \int_a^b g \, d\gamma.$

(B) $\int_a^b (\alpha f + \beta g) d\gamma = \alpha \int_a^b f \, d\gamma - \beta \int_a^b g \, d\gamma.$

(C) $\int_a^b (\alpha f + \beta g) d\gamma = |\alpha| \int_a^b f \, d\gamma + |\beta| \int_a^b g \, d\gamma.$

(D) $\int_a^b (\alpha f + \beta g) d\gamma = \alpha \int_a^b f \, d\gamma + \beta \int_a^b g \, d\gamma.$

15. If $\gamma : [a,b] \to \mathbb{C}$ is a rectifiable path and $\varphi : [c,d] \to [a,b]$ is a continuous non-decreasing function with $\varphi(c) = a, \varphi(d) = b$; then for any function f continuous on $\{\gamma\}$ then ————.

(A) $\int\limits_{\gamma} f = \int\limits_{\gamma \circ \varphi} f.$ (B) $\int\limits_{\gamma} f = \int\limits_{\varphi \circ \gamma} f.$

(C) $\int\limits_{\gamma} f = \int\limits_{\varphi} f.$ (D) None of the above options.

16. Let γ be a rectifiable curve and suppose that f is a function continuous on $\{\gamma\}$. Then ————.

(A) $\int\limits_{\gamma} |f|\,|dz| \le \left|\int\limits_{\gamma} f\right| \le V(\gamma) \sup\{|f(z)| : z \in \{\gamma\}\}.$

(B) $\left|\int\limits_{\gamma} f\right| \ge \int\limits_{\gamma} |f|\,|dz| \ge V(\gamma) \sup\{|f(z)| : z \in \{\gamma\}\}.$

(C) $\left|\int\limits_{\gamma} f\right| = \int\limits_{\gamma} |f|\,|dz| = V(\gamma) \sup\{|f(z)| : z \in \{\gamma\}\}.$

(D) $\left|\int\limits_{\gamma} f\right| \le \int\limits_{\gamma} |f|\,|dz| \le V(\gamma) \sup\{|f(z)| : z \in \{\gamma\}\}.$

17. If $|z| < 1$ then $\int\limits_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds =$ ————.

(A) 2π. (B) -2π.

(C) $2\pi i$. (D) $-\pi$.

18. If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point $a \in G$ with $|f(a)| \ge |f(z)|$ for all $z \in G$, then ————.

(A) f is constant. (B) f is not a constant.

(C) f is a real number. (D) f is a purely imaginary number.

Turn over

19. Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If $\gamma_1, \gamma_2, \ldots, \gamma_m$ are closed rectifiable curves in G such that

$$n(\gamma_1; w) + n(\gamma_2; w) + \ldots + n(\gamma_n; w) = 0 \text{ for all } w \in \mathbb{C} \sim G,$$

then for $a \in G \sim \bigcup_{k=1}^{m} \{\gamma_k\}$ _____.

(A) $\sum_{k=1}^{m} n(\gamma_k; a) = \sum_{k=1}^{m} \frac{1}{2\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

(B) $f(a) \sum_{k=1}^{m} n(\gamma_k; a) = \sum_{k=1}^{m} \frac{1}{2\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

(C) $f(a) \sum_{k=1}^{m} n(\gamma_k; a) = \sum_{k=1}^{m} \frac{1}{\pi i} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

(D) $f(a) \sum_{k=1}^{m} n(\gamma_k; a) = \sum_{k=1}^{m} \frac{1}{2\pi} \int_{\gamma_k} \frac{f(z)}{z-a} dz.$

20. If $f : G \to \mathbb{C}$ is an anaytic function and γ is a closed rectifiable curve in G such that $\gamma \sim 0$, then $\int_{\gamma} f =$ _____.

(A) 1. (B) π.

(C) 2π. (D) 0.