D 52826 **(Pages : 3)** **Name......................................**

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define strict partial order. Give an example.
2. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that for all elements x and y of X,

   $x + x \cdot y = x$.
3. Give an example of a Boolean function.
4. Define complete graph.
5. Prove that for any simple graph G, $Aut(G) = Aut(G^c)$.
6. If G is a simple planar graph with at least three vertices, then prove that $m \leq 3n - 6$.
7. Give an example of a dfa.
8. Define language accepted by an nfa.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Define a chain in a poset. Prove that the intersection of two chains is a chain.

10. Write the following Boolean function in their disjunctive normal form.

$$f(x_1, x_2, x_3) = (x_1 + x_2')\, x_3' + x_2 x_1' (x_2 + x_1' x_3').$$

11. State and prove law of uniqueness of complements in a Boolean algebra.

MODULE II

12. Prove that the number of edges of a simple graph of order n having ω components cannot exceed-

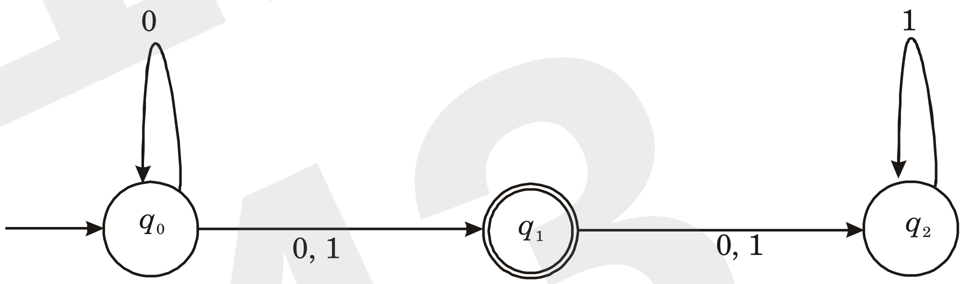
$$\frac{(n-\omega)(n-\omega+1)}{2}.$$

13. Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of G.

14. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G if and only if there exist vertices u and w of G distinct from v such that v is in every u-w path in G.

MODULE III

15. Convert the nfa in the following figure into an equivalent deterministic machine.



16. Find a deterministic finite accepter that recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix ab.

17. Show that the language

$$L = \left\{awa : w \in \{a, b\}^{*}\right\}$$

is regular.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that the corresponding lattice $(X, \leq)$ is complemented and distributive. Conversely prove that if $(X, \leq)$ is bounded, complemented and distributive lattice then there exists a Boolean algebra structure on X. $(X, +, \cdot, ')$ such that the partial order relation defined by this structure coincides with the given relation $\leq$.

19. Prove that a graph is bipartite if and only if it contains no odd cycles.

20. Prove that the set $Aut(G)$ of all automorphisms of a simple graph G is a group with respect to the composition $\circ$ of mappings as the group operation.

21. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite accepter, and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$, and $w \in \sum^{+}$, $\delta^{*}(q_j, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j

(2 × 5 = 10 weightage)