D 52827 **(Pages : 3)** **Name.....**

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Prove that the power function $N_\alpha(n) = n^\alpha$, where α is a fixed real or complex number is completely multiplicative.
2. State Generalized inversion formula.
3. Prove that $\sum_{n>x} \frac{1}{n^s} = O\left(x^{1-s}\right)$ if $s > 1$.
4. For $x \geq 2$, show that $\pi(x) = \frac{\vartheta(x)}{\log x} + \int_2^x \frac{\vartheta(t)}{t \log^2 t}\, dt$.
5. Describe briefly about RSA cryptosystems.
6. Define Legendre's symbol and evaluate the Legendre's symbol $(-1|p)$.
7. State Reciprocity law for Jacobi symbol.
8. Determine whether 888 is a quadratic residue or nonresidue of the prime 1999.

(8 × 1 = 8 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. For $n \geq 1$ show that $\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.

10. Let f be a completely multiplicatively function. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n), \forall n \geq 1$.

11. For $x \geq 1$, show that $\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \zeta(s) + O\left(x^{-s}\right)$, if $s > 0, s \neq 1$.

Module II

12. For all $x \geq 1$, show that $\sum_{p \leq x} \frac{\log p}{p} = \log x + O(1)$.

13. State and prove Abel's identity.

14. Let $\{a_n\}$ be a non negative sequence such that $\sum_{n \leq x} a(n) \left[\frac{x}{n}\right] = x \log x + O(x)$ for all $x \geq 1$. Prove that $\sum_{n \leq x} \frac{a(n)}{n} = \log x + O(1)$.

Module III

15. Sate and prove Euler's criterion.

16. If P is positive odd integer prove that Jacobi symbol $(-1|P) = (-1)^{(P-1)/2}$ and $(2|P) = (-1)^{(P^2-1)/8}$.

17. How do classical and public cryptosystem differ ?

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. (a) If g and $f * g$ are multiplicative prove that f is multiplicative.

    (b) If f is multiplicative prove that $\sum_{d/n} \mu(d) f(d) = \prod_{p/n} (1 - f(p))$.

19. State and prove Euler's summation formula.

20. For every integer $n \geq 1$, prove that the n^{th} prime p_n satisfies the inequalities

$$\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right).$$

21. State and prove Quadratic Reciprocity law for Legendre's symbol.

(2 × 5 = 10 weightage)