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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2023 1244 (Pages : 5)
 12444 (Pages : 5)
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 12444 EXAMINATION, APRIL 2023

(CBCSS)

Mathematics

TH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

Maximum : 30 Weightage

(CBCSS)

Mathematics

MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer all questions. Each question carries 1 *weightage.*

- 1. Give confluent hypergeometric equation.
- 2. Find the first three terms of the Legendre series of

$$
f(x) = \begin{cases} 0 & \text{if } -1 \leq x < 0, \\ x & \text{if } 0 \leq x \leq 1. \end{cases}
$$

3. Determine whether the following functions if positive definite, negative definite, or neither : **11.** Three Hours
 11. Answer **all** questions.
 11. Give confluent hypergeometric equation.

2. Find the first three terms of the Legendre series of
 $f(x) = \begin{cases} 0 & \text{if } -1 \le x < 0, \\ x & \text{if } 0 \le x \le 1. \end{cases}$

3. Determine w

 $-2x^2 + 3xy - y^2$.

4. Show that $(0, 0)$ is a simple critical point of the system :

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\n
$$
\begin{cases}\n\frac{dx}{dt} = -2x + 3y + xy \\
\frac{dy}{dt} = -x + y - 2xy^2\n\end{cases}
$$
\n5. Describe the phase portrait of the system
\n
$$
\begin{cases}\n\frac{dx}{dt} = x \\
\frac{dy}{dt} = 0\n\end{cases}
$$
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5. Describe the phase portrait of the system

$$
\begin{cases}\n\frac{dx}{dt} = x \\
\frac{dy}{dt} = 0\n\end{cases}
$$
\n**Turn over**

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- 6. Show that $f(x, y) = xy^2$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$.
- 7. Describe Picard's iteration method.
- 8. Give the solution, if exists, to the initial value problem

$$
y' = t, y(0) = 1.
$$

 $(8 \times 1 = 8$ weightage)

Part B

Answer any **two** *questions from each of the following* **three** *units. Each question carries* 2 *weightage.* **382987**

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11431 $y = xy^2$ satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$.

12431 Tailors method.

13431 Tailors method.

13431 $y' = t$, $y(0) = 1$.

23431 Tailors method.

1444 Tailors met Answer any **two** questions from each of the following **three** units.

Each question carries 2 weightage.

UNIT I

9. Express $\sin^{-1} x$ in the form of a power series by solving the equation
 $y' = (1 - x^2)^{-1/2}$, $y(0) = 0$

in

UNIT I

9. Express $\sin^{-1} x$ in the form of a power series by solving the equation

$$
y' = \left(1 - x^2\right)^{-1/2}, \, y(0) = 0
$$

in two ways.

10. Locate and classify singular points on the *x*-axis of the differential equation

$$
x^{2}(x^{2}-1)^{2}y''-x(1-x)y'+2y=0.
$$

11. Find the general solution of the differential equation

$$
(2x^{2}+2x)y''+(1+5x)y'+y=0
$$
near the singular point $x=0$.
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near the singular point $x = 0$.

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UNIT II

12. If the two solutions

 (t) (t) (t) (t) 1 $\binom{1}{2}$ $\binom{3}{2}$ 1 (*i*) $y - y_2$ and $x = x_1(t)$ $\qquad \qquad$ $x = x_2(t)$ $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ **382987**

UNIT II

tions
 $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$

eous system
 $\begin{cases} \frac{dx}{dt} = a_1(t) x + b_1(t) y \\ \frac{dy}{dt} = a_2(t) x + b_2(t) y \end{cases}$

of the homogeneous system

 $(t) x + b_1 (t)$ $(t) x + b_2 (t)$ $1^{(\iota) \cdot \iota + o_1}$ $2(\iota)x+\nu_2$ $\frac{dx}{dt} = a_1(t)x + b_1(t)y$ *dt* $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ *dt* $\int \frac{dx}{dt} = a_1(t) x +$ $\frac{dy}{dt} = a_2(t)x +$

are linearly independent on $[a, b]$, then prove that

$$
\begin{cases} x = c_1 x_1 (t) + c_2 x_2 (t) \\ y = c_1 y_1 (t) + c_2 y_2 (t) \end{cases}
$$

is the general solution of the homogeneous system on this interval.

- 13. Prove that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$, and is negative definite if and only if $a < 0$ and $b^2 - 4ac < 0$. $\left|\frac{dx}{dt} = a_1(t)x + b_1(t)y\right|$ $\left|\frac{dy}{dt}\right| = a_2(t)x + b_2(t)y$ are linearly independent on [a, b], then prove that
 $\left\{x = c_1x_1(t) + c_2x_2(t)\right.\\ \left\{y = c_1y_1(t) + c_2y_2(t)\right.\right.\\$ is the general solution of the homogeneous system on this
- 14. Determine the nature and stability properties of the critical point $(0, 0)$ for the following linear autonomous system : 14. Determine the nature and stability properties of the critical point
autonomous system :
 $\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 8x - 6y \end{cases}$
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$$
\begin{cases}\n\frac{dx}{dt} = 4x - 3y \\
\frac{dy}{dt} = 8x - 6y\n\end{cases}
$$

Turn over

UNIT III

- 15. Let $y_p(x)$ be a non-trivial solution of Bessel's equation on the positive *x*-axis. If $0 \le p < 1/2$, then prove that every interval of length π contains at least one zero of $y_p(x)$; if $p = 1/2$, then prove that the distance between successive zeros of $y_p(x)$ is exactly π ; and if $p > 1/2$, then prove that α every interval of length π contains at most one zero of $y_p(x).$ **382987**

UNIT III

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y interval of length π contains at least one zero of $y_p(x)$; if $p = 1/2$, then prove

ce between successive
- 16. Given $\frac{dy}{dx} = x + y$ $\frac{dy}{dx}$ = $x + y$ with the initial condition *y* (0) = 1. By Picard's iteration method, find approximate value of *y* for $x = 0.2$ and $x = 1$. 16. Given $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$. By Picard's iteration method, find approv
value of y for $x = 0.2$ and $x = 1$.
17. Prove that the geodesics on a sphere are arcs of great circles.
Part C
Answ
- 17. Prove that the geodesics on a sphere are arcs of great circles.

 $(6 \times 2 = 12$ weightage)

Part C

Answer any **two** *questions. Each question carries* 5 *weightage.*

18. (a) Find a power series solution of the form $\sum a_n x^n$ of the differential equation

 $y' + y = 1.$

(b) Show that equation

$$
4x^2y'' - 8x^2y' + (4x^2 + 1) y = 0
$$

has only one Frobenius series solution and find it.

19. (a) Derive Rodrigue's formula for Legendre polynomials

$$
4x^{2}y'' - 8x^{2}y' + (4x^{2} + 1) y = 0
$$

has only one Frobenius series solution and find it.
19. (a) Derive Rodrigue's formula for Legendre polynomials

$$
P_{n}(x) = \frac{1}{2^{n}n!} \frac{d^{n}}{dx^{n}} (x^{2} - 1)^{n}.
$$

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(b) If the roots m_1 and m_2 of

$$
m^2 - (a_1 + b_2) m + (a_1b_2 - a_2b_1) = 0
$$

are real, distinct, and of the same sign, then prove that the critical point $(0, 0)$ of the system **382987**
 m_1 and m_2 of
 $m^2 - (a_1 + b_2) m + (a_1b_2 - a_2b_1) = 0$

istinct, and of the same sign, then prove that the critical point $(0, 0)$ of the system
 $\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}$

$$
\begin{cases}\n\frac{dx}{dt} = a_1 x + b_1 y \\
\frac{dy}{dt} = a_2 x + b_2 y\n\end{cases}
$$

is a node.

20. (a) If $q(x) < 0$, and if $u(x)$ is a nontrivial solution of **123**

is a node.

20. (a) If $q(x) < 0$, and if $u(x)$ is a nontrivial solution of
 $u'' + q(x)u = 0$

then prove that $u(x)$ has at most one zero.

(b) For the following nonlinear system :

i) Find the critical points;

ii) Find

$$
u'' + q(x) u = 0
$$

then prove that $u(x)$ has at most one zero.

- (b) For the following nonlinear system :
	- i) Find the critical points ;
	- ii) Find the differential equation of the paths ; and
	- iii) Solve this equation to find the paths

$$
\begin{cases}\n\frac{dx}{dt} = e^y \\
\frac{dy}{dt} = e^y \cos x\n\end{cases}
$$

- 21. (a) Find the curve joining two points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about the *x*-axis. $\frac{dx}{dt} = e^y$
 $\frac{dy}{dt} = e^y \cos x$

21. (a) Find the curve joining two points (x_1, y_1) and (x_2, y_2) that yields minimum area when revolved about the *x*-axis.

(b) State and prove Picard's theorem.
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	- (b) State and prove Picard's theorem.

 $(2 \times 5 = 10$ weightage)