C 42791 (Pages : 3) Name.......................................

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2023

(CBCSS)

Mathematics

MTH 2C 10—OPERATIONS RESEARCH

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

*Answer **all** questions.*
Each question carries 1 weightage.

1. Prove that $f(x) = 2x_1^2 + 2x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$ is a convex function.
2. Show that the set S_F of feasible solutions, if not empty, is a closed convex set bounded from below and so has at least one vertex.
3. Define the dual of an LP problem. Illustrate with an example.
4. Write the important steps to setting up the mathematical model for a linear programming problem.
5. State few applications of duality.
6. Solve the game whose payoff matrix is $\begin{pmatrix} 15 & 16 \\ 20 & 5 \end{pmatrix}$.
7. Find the optimal strategies and the value of the game : $\begin{pmatrix} -5 & 31 & 20 \\ 5 & 54 & 6 \\ -4 & -20 & -5 \end{pmatrix}$.
8. State any one disadvantage of Cutting plane method.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** *questions from each of the following* **three** *units.*
Each question carries 2 *weightage.*

Unit I

9. Explain the method of solving a linear programming problem with two variables using graphical method.

10. Let $f(\mathrm{X})$ be defined in a convex domain $\mathrm{K} \subseteq \mathrm{E}_n$ and be differentiable. Derive a necessary and sufficient condition for $f(\mathrm{X})$ to be a convex function.

11. Write a sequence of steps that constitutes one iteration leading from one basic feasible solution to another.

Unit II

12. How do we test for optimality while solving a transportation problem.

13. Show that the optimum value of $f(\mathrm{X})$ of the primal, if it exists, is equal to the optimum value of $\phi(\mathrm{Y})$ of the dual.

14. Characterize any set of linearly dependent column vectors P_{ij} in the matrix $\bar{\mathrm{T}}$.

Unit III

15. Show that the problem of solving a rectangular game is equivalent to solving a problem of linear programming.

16. Discuss on the introduction of new constraints while determining a new optimal solution from the optimal solution already obtained.

17. Explain the maximum flow problem.

(6 × 2 = 12 weightage)

Part C

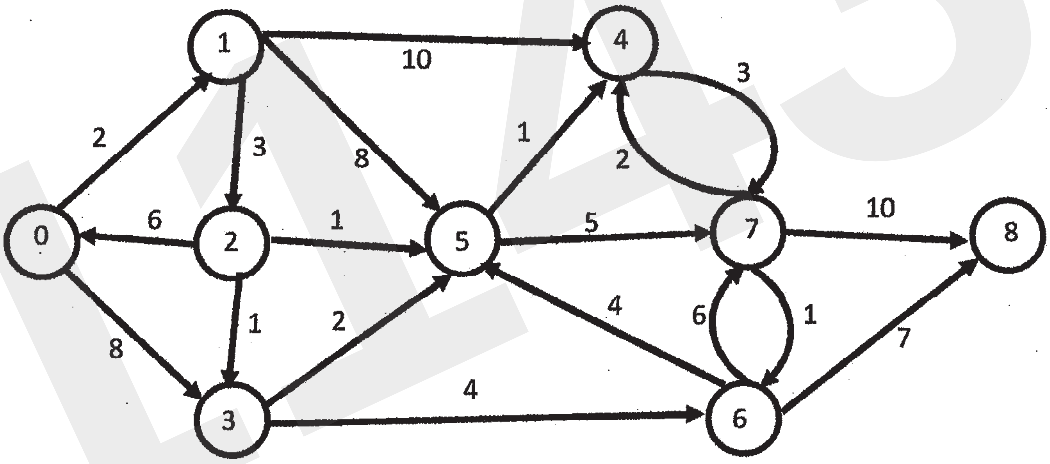
Answer any **two** *questions.*
Each question carries 5 weightage.

18. If S_F is non-empty, then show that the objective function $f(X)$ has either an unbounded minimum or it is minimum at a vertex of S_F.

19. Use dual simplex method to solve the following LP problem :

    Minimize $f = 3x_1 + 5x_2 + 2x_3$ subject to $-x_1 + 2x_2 + 2x_3 \geq 3$; $x_1 + 2x_2 + x_3 \geq 2$; $-2x_1 - x_2 + 2x_3 \geq -4$, $x_1, x_2, x_3 \geq 0$.

20. Find the minimum path from v_0 to v_8 in the graph given below in which the number along a directed arc denotes its length.



21. Explain the Branch and Bound method with an example.

(2 × 5 = 10 weightage)