D 51311 **(Pages : 3)** **Name..................................**

Reg. No..............................

THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each questions carries a weightage of 1.

1. Find the general solution to the equation $-yu_x + xu_y = 0$.
2. Check whether the transversality condition holds for the Cauchy problem :

$$xu_x - yu_y = u + xy,\ u(x,x) = x^2, 1 \le x \le 2.$$

3. Describe Rankine-Hugoniot condition.
4. Prove the uniqueness of heat conduction problems with Dirichlet boundary condition.
5. Describe Duhamel's principle.
6. Show that the solution of the Neumann problem differ by constant in a smooth domain.
7. State four properties of Green's function.
8. Find the resolvent kernel of the Volterra integral equations with the kernel $K(x,\xi) = e^{x^2-\xi^2}$.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit.*
Each question carries a weightage of 2.

Unit I

9. Show the equation $u_x + 3y^{\frac{2}{3}}u_y = 2$ subject to the initial condition $u(x,1) = 1 + x$.
10. Find the canonical form and general solution of the differential equation :

$$u_{xx} - 2\sin(x)u_{xy} - \cos(2x)u_{yy} - \cos(x)u_y = 0.$$

11. Explain and justify the wellposedness of Cauchy problem for the one-dimensional homogeneous wave equation.

Unit II

12. Solve $u_t - u_{xx} = 0, 0 < x < \pi, t > 0$

$$u(0,t) = u(\pi,t) = 0, t \geq 0$$

$$u(x,0) = f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi. \end{cases}$$

13. State and prove the mean value principle.

14. Find the harmonic function in the unit square satisfying the Dirichlet conditions $u(x,0) = 1 + \sin(\pi x), u(x,1) = 2, u(0,y) = u(1,y) = 1 + y.$

Unit III

15. Construct Green's function for the boundary value problem :

$$y'' = 0; y(0) = y(l).$$

16. Find the eigenvalues and eigenfunctions of the homogeneous integral equations :

$$y(x) = \lambda \int_0^1 \left(2x\xi - 4x^2\right) y(\xi) d\xi.$$

17. Show that the eigenfunctions of a symmetric kernel, corresponding to different eigenvalue are orthogonal.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. State and prove the existence theorem for quasilinear first order partial differential equations.

19. For the problem $u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0$ with initial conditions :

$$u(x,0) = f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

$$u_t(x,0) = \begin{cases} 4, & 1 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $u(x, 1)$.

(b) Find $\lim_{t\to\infty} u(5,t)$.

(c) Find the set of all points where the solution is singular.

(d) Find the set of all points where the solution is continuous.

20. Derive Poisson's formula.

21. Determine the resolvent kernel of $y(x) = 1 + \lambda \int_0^1 (1-3\xi)\, y(\xi)\, d\xi$ where $k(x,\xi) = 1 - 3x\xi$ for what value of λ the solution does not exists. Obtain the solution of the above integral equation.

(2 × 5 = 10 weightage)