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D 51315 **(Pages : 3)** **Name....................................**

Reg. No.................................

THIRD SEMESTER M.Sc. (CBCSS) [REGULAR/SUPPLEMENTARY] DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each questions carries a weightage of 1.

1. If X is a random variable, show that $a\mathrm{X} + b$ is also a random variable.
2. If $g(\mathrm{X})$, is a non-negative Borel function of the r.v. X, then prove that

$$\mathrm{P}\left(g(\mathrm{X}) \geq \in\right) \leq \frac{\mathrm{E}\left(g(\mathrm{X})\right)}{\in}, \text{ if } \mathrm{E}\left(g(\mathrm{X})\right) < \infty.$$

3. State the Jensen's inequality, mentioning the necessary conditions.
4. For a r.v. X if $\mu_r' < \infty$ then prove that $\mu_s' < \infty$, for $0 < s < r$; r & s are integers.
5. Define conditional expectation of random variables.
6. If $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_n$ are iid random variables having uniform distribution in $(0, \theta)$, obtain the marginal p.d.f of $\mathrm{X}_{(1)}$ and $\mathrm{X}_{(n)}$.
7. Say True or False. $\mathrm{X}_n \xrightarrow{a.s} \mathrm{X} \leftrightarrow \lim\limits_{n\to\infty} \mathrm{P}\left[\sup\limits_{m\geq n}\left|\mathrm{X}_m - \mathrm{X}\right| \geq \in\right] = 0, \forall \in > 0.$
8. Examine whether the WLLN holds for the sequence of independent rvs $\{\mathrm{X}_n, n \geq 1\}$ distributed as $\mathrm{P}\left[\mathrm{X}_n = \pm 2n\right] = 2^{-(2n+1)}, \mathrm{P}\left[\mathrm{X}_n = 0\right] = 1 - 2^{-2n}$.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions choosing two from each unit.*
Each question carries a weightage of 2.

Unit I

9. Prove that X is a non-negative r.v. having distribution function $\mathrm{F_X}(x)$, with $\mathrm{E(X)} < \infty$, if and only if $\int_0^\infty \left(1 - \mathrm{F_X}(x) dx\right) < \infty$ and then $\mathrm{E(X)} = \int_0^\infty \left(1 - \mathrm{F_X}(x)\right) dx$.

Turn over

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10. Show that the characteristic function $\phi(t)$ of a r.v is uniformly continuous on R. Also examine whether $\phi(t) = \log(1+t)$ is a characteristic function.

11. State and prove the Hölder's inequality, mentioning the assumptions to be satisfied. When will it reduce to the Cauchy-Schwartz inequality ?

Unit II

12. Show that $\mu^2_{X,Y} \le \sigma^2_X \sigma^2_Y$, where $\mu^2_{X,Y}$ is the bivariate central moment of order 2 of the random vector (X, Y).

13. Establish the linearity property of conditional expectation.

14. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics from a population with absolutely continuous distribution function F. Derive the joint pdf of $X_{(r)}$ and $X_{(s)}$.

Unit III

15. Prove that : $X_n \xrightarrow{r} X \rightarrow X_n \xrightarrow{P} X$. When is the converse true ? Use the following example to prove it. Let $\{X_n, n \ge 1\}$ be a sequence of r.vs such that $P(X_n = 0) = 1 - \frac{1}{n}$, $P\left(X_n = e^n\right) = \frac{1}{n}, n \ge 1$.

16. Consider the sequence of r.vs $\{X_i\}$ of independent r.vs with $P\left[X_i = \frac{i}{\log i}\right] = P\left[X_i = \frac{-i}{\log i}\right] = \frac{\log i}{2i}$ and $P[X_n = 0] = 1 - \frac{\log i}{i}$. Examine if the sequence obeys WLLN. If not, mention when will it obey the WLLN.

17. (i) State and prove the Lindberg-Levy CLT.

    (ii) Show that it is a special case of the Lindberge-Feller form of CLT.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Let $P\left[X = 2^x\right] = \frac{e^{-1}}{x!}, x = 0, 1, 2, \ldots,$ be the pmf of X. Find the MGF. Hence or otherwise obtain the moments.

19. The joint pdf of two random variables X and Y is :

$$f(x, y) = \begin{cases} 3(x+y); & 0 \le x, y \le 1 \\ 0; & \text{otherwise} \end{cases}$$

Find (1) the marginal pdfs of X and Y, (2) the conditional pdfs, (3) conditional means, (4) conditional variances (5) Cov (X, Y) and the correlation co-efficient between X and Y.

20. If X and Y are independent rectangular varieties on [0, 1], find the distribution of (i) X + Y, (ii) X – Y, (iii) $|X - Y|$.

21. Test whether for sequence $\{X_n\}$ of independent r.vs with $P\left[X_n = \pm 2^n\right] = \frac{1}{2}$, obey the WLLN and SLLN.

(2 × 5 = 10 weightage)

D 51315-A **(Pages : 5)** **Name.....................................**

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THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTH 3E 04—PROBABILITY THEORY

(2019 Admission onwards)

(Multiple Choice Questions for SDE Candidates)

[Improvement Candidates need not appear for MCQ part]

Time : 20 Minutes	**Total No. of Questions : 20**	**Maximum : 5 Weightage**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTH 3E 04—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. If X and Y are two random variables with means $\overline{X}$ and $\overline{Y}$ respectively, then the expression $E\left[\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)\right]$ is called :

   (A) Variance of X. (B) Variance of Y.

   (C) Cov (X, Y). (D) Moments of X and Y.

2. The random variables X and Y have variances 0.2 and 0.5 respectively. Let Z = 5X – 2Y. The variance of $\mathbb{Z}$ is :

   (A) 3. (B) 7.

   (C) 4. (D) 5.

3. The weight of persons in a country is a r.v. of the type :

   (A) Continuous r.v.

   (B) Discrete r.v.

   (C) Neither discrete nor continuous.

   (D) Discrete as well as continuous.

4. If X is a random variable, $E\left(e^{itx}\right)$ is known as :

   (A) Characteristic function.

   (B) Moment generating function.

   (C) Probability generating function.

   (D) All the above.

5. In the continuous case, P {X = a} is :

   (A) 1. (B) ∞.

   (C) 0. (D) None of the above.

6. Consider a r.v X that takes values +1 and – 1 with probability 0.5 each. The values of the distribution function F(x) at x = – 1 and + 1 are :

   (A) 0 and 0.5. (B) 0 and 1.

   (C) 0.5 and 1. (D) 0.25 and 0.75.

7. Two random variables X and Y are said to be independent if :

   (A) E(XY) = 1.
   (B) E(XY) = 0.
   (C) E(XY) = E(X)E(Y).
   (D) E(XY) = any constant value.

8. A discrete r.v has probabilty mass function $p(x) = kq^x p,\ p + q = 1,\ x = 2, 3, 4, \ldots$ the value of k should be equal to :

   (A) $1/q^2$.
   (B) $1/p$.
   (C) $1/q$.
   (D) $1/pq$.

9. If X is a random variable which can take only non-negative values, then :

   (A) $E\left(X^2\right) = [E(X)]^2$.
   (B) $E\left(X^2\right) \geq [E(X)]^2$.
   (C) $E\left(X^2\right) \leq [E(X)]^2$.
   (D) None of the above.

10. A r.v is a ———— function.

   (A) Continuous.
   (B) Discrete.
   (C) Real valued.
   (D) None of the above.

11. The outcomes of tossing a coi three times are a variable of the type :

   (A) Continuous r.v.
   (B) Discrete r.v.
   (C) Neither discrete nor continuous.
   (D) Discrete as well as continuous.

12. If X is a continuous r.v., $P(a \leq x \leq b)$ ————.

   (A) $F(a) - F(b)$.
   (B) $F(a) * F(b)$.
   (C) $F(b) - F(a)$.
   (D) None of the abve.

13. A continuous r.v X has pdf $f(x) = kx,\ 0 < x < 1,$ the value of k = ————.

   (A) 3.
   (B) 2.
   (C) 1.
   (D) 4.

14. If $V(x) = 1$, then $V(2x \pm 3)$ is :

   (A) 5.
   (B) 13.
   (C) 4.
   (D) 10.

15. If X and Y are independent, then Cov (X, Y) = ————.

(A) 1. (B) 0.

(C) 2. (D) None of the above.

16. Let (X, Y) be an RV of the discrete type, then conditional PMF of X, given Y = y_j defined as :

(A) $P\{Y = y_j | X = x_i\} = \frac{P(X = x_i, Y = y_j)}{P\{Y = y_j\}}$.

(B) $P\{Y = y_j | X = x_i\} = \frac{P(X = x_i, Y = y_j)}{P\{X = x_i\}}$.

(C) $P\{X = x_i | Y = y_j\} = \frac{P(X = x_i, Y = y_j)}{P\{Y = y_j\}}$.

(D) $P\{X = x_i | Y = y_j\} = \frac{P(X = x_i, Y = y_j)}{P\{X = x_i\}}$.

17. Let (X, Y) be an RV of the continuous type, then conditional PDF of X, given Y = y defined as :

(A) $f_{X/Y}(x | y) = \frac{f(x, y)}{f_2(y)}$. (B) $f_{Y/X}(y | x) = \frac{f(x, y)}{f_2(y)}$.

(C) $f_{X/Y}(x | y) = \frac{f(x, y)}{f_1(x)}$. (D) $f_{Y/X}(y | x) = \frac{f(x, y)}{f_1(x)}$.

18. If Var(Y) = 6, then Var (2Y–4) = ————.

(A) 8. (B) 16.

(C) 20. (D) 24.

19. For any real numbers a and b, $a \leq b$, the probability density function of a continuous RV X is given by ————.

(A) $P\{a \leq X \leq b\} = \int_a^b f(x)dx$. (B) $P\{a \geq X \geq b\} = \int_a^b f(x)dx$.

(C) (a) $P\{a \leq X \leq b\} = 1 - \int_a^b f(x)dx$. (D) (a) $P\{a \leq X \leq b\} = \int_a^b f(x)dx - 1$.

20. For a given probability distribution

$$f(x) = \frac{1}{8} 3C_x, x = 0, 1, 2, 3,$$

the moment generating function of X is ———.

(A) $\frac{1}{8}\left(1+e^t\right)^3$. (B) $\frac{1}{4} e^t$.

(C) $\left(1+e^t\right)^2$. (D) e^t.