D 52838 **(Pages : 2)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Physics

PHY IC 02—MATHEMATICAL PHYSICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Section A

8 *short questions answerable within* 7.5 *minutes.*
Answer **all** *questions, each question carries* 1 *weightage.*

1. Write down the relation between cartesian coordinate system and spherical polar co-ordinate system.
2. What are Tensors ? Define the rank of the tensor.
3. Write the expression for Fourier co-efficients.
4. What do you mean by a self-adjoint differential equation ?
5. Write down the Rodrigues formula of Laguerre function and obtain $L_1(x)$ from Rodrigues formula.
6. Define a unitaiy matrix with an example.
7. Obtain the recurrence formula, $H'_n(x) = 2nH_{n-1}(x)$ from generating function.
8. Explain the convolution property of Fourier transform with an example.

(8 × 1 = 8 weightage)

Section B

4 *essay questions answerable within* 30 *minutes.*
Answer any **two** *questions, each question carries weightage* 5.

9. Discuss the orthogonality property of Legendre polynomials.
10. Explainthe Frobenius' method of finding solution to homogenous differential equation of second order.

Turn over

11. Prove that $\nabla \cdot r^n \hat{r} = (n+2) r^{n-1}$.

12. State and prove the Quotient rule in tensors.

(2 × 5 = 10 weightage)

Section C

7 problems answerable within 15 minutes.
Answer any **four** *questions, each question carries weightage* 3.

13. Express the spherical polar unit vectors in terms of cartesian unit vectors.

14. Show that $\Gamma(p+1) = p\,\Gamma(p)$.

15. Prove that $H_{2n}(0) = (-1)^n \dfrac{(2n)!}{n!}$.

16. Find Laplace transform of the function, $f(t) = t^n$.

17. Define spherical Bessel function. Obtain the expression for $j_1(x)$.

18. Find the Fourier transform of the normalized Gaussian distribution

$$f(t) = \frac{1}{\tau\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\tau^2}\right), -\infty < t < \infty, \text{ where } \tau = \Delta t$$ (root mean square deviation).

19. A and B are two non-commuting Hermitian matrices : $AB - BA = iC$. Prove that C is Hermitian.

(4 × 3 = 12 weightage)