Q.P Code	D 112645	Total Pages: 2	Name
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FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2024

(CUFYUGP)

MAT1MN102 - CALCULUS OF A SINGLE VARIABLE

2024 Admissions

Maximum Time :2 Hours	**Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	Does $\lim_{x\to 0^+} \frac{1}{x} = \lim_{x\to 0^-} \frac{1}{x}$ exist? Explain.
2	Find $\lim_{x\to 3^+} \frac{x}{x-3}$
3	Does the function $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ x+10 & \text{if } x > 2 \end{cases}$, continuous at 2? Why?
4	During the first 30 s of a rocket flight, the rocket is pro- pelled straight up so that in t seconds it reaches a height of $s = t^2 + 2$ ft. What is the average velocity of the rocket during the first 1000 ft of its flight?
5	Using the definition of differentiation find $\frac{d}{dx}[3x]$
6	Find $\frac{dy}{dx}$ if $y = \log_{25}(x^2 - 1)$
7	Find $\lim_{x\to 0^+} \frac{\tan x}{x}$, $x \in (0, \pi/2)$
8	Use logarithmic differentiation to find $\frac{d}{dx}[2^{\cos x + \ln x}]$
9	Explain a increasing function with an example
10	Find all critical points of $f(x) = x^3 - 3x + 1$

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))

11	Find $\lim_{x\to+\infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
12	Show that $\|x\|$ is continuous everywhere
13	Let $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 4\sqrt{x} & \text{if } x > 1 \end{cases}$ Determine whether f is differentiable at x = 1. If so, find the value of the derivative there
14	Find $\dfrac{dy}{dx}$, if $y = \sin(\sqrt{1+\tan x})$
15	Use implicit differentiation to find all points on the graph of $y^4 + y^2 = x(x-1)$ at which the tangent line is vertical.
16	Evaluate $\lim_{x\to 0^+} x^3 \ln x \;\; ; x \in (0, \infty)$
17	Find the inflection points of the function $f(x) = x^3 - 3x^2 + 1$
18	Find the relative extrema of $f(x) = \dfrac{x^2}{x^4 + 16}$

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19	On a sunny day, a 50 ft flagpole casts a shadow that changes with the angle of elevation of the Sun. Let s be the length of the shadow and θ the angle of elevation of the Sun. Find the rate at which the length of the shadow is changing with respect to θ when $\theta = 45°$.
20	For the function $f(x) = x^4 - 5x^3 + 9x^2$ 1. Find the intervals on which f is increasing and decreasing 2. Find the open intervals on which f is concave up and concave down 3. Find all of its inflection points, if any.